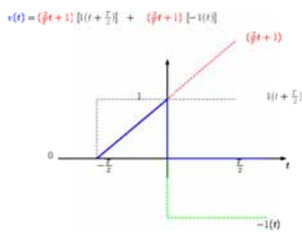
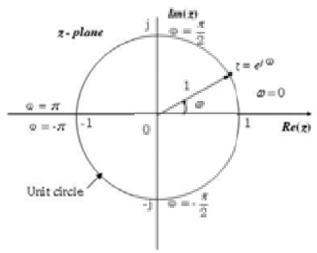
SIGNALs & SYSTEMs

TE-301

LECTURE NOTES





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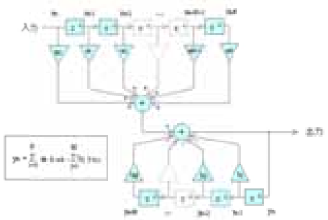
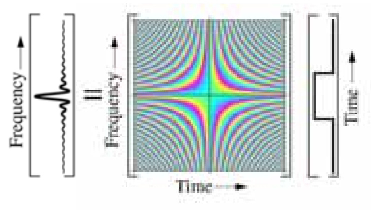




(TE 301)

Signals & Systems

(...07)

Text book:

* ① Continuous and discrete signals & systems. By: Samir S Soliman & Mandyam D. Srinath (2nd ed)
  ② Signals & systems (2nd ed) by: Allan V. Oppenheim & Allan Willsky.

Signal:

① The detectable Physical Quantities or variables by means of which information or message can be transmitted.

(OR)

② Signal is an abstraction of any measurable quantity i.e. fns of one or more independent ~~of~~ e.g. time.

System:

① A system is process that results in transforming the i/p signals into o/p signals.

(OR)

② A system is an abstraction of any thing takes i/p and operates on it & produces an o/p signal.

Signal & System :-

It is the study of signals & their interaction with systems. it is fundation of all advanced engineering analysis & measurements. it provides a variety of tools 'n' techniques.

Classification of Signals :-

① Continuous / Discrete Signals :

A signal $x(t)$ w/c fns of time is a continous signal if the independant variable t is continous over a total interval. If independant variable is discrete is called discrete time signal.

A discrete time signal is often identified as a sequence of no.s denoted by $\{x_n\}$ or $x[n]$ where 'n' is an integar.

② Digital / Analog Signals

If a C-T signal $x(t)$ can take on any value in continous interval (a, b) where 'a' may be $-ve$ ∞ and 'b' may $+\infty$. then C-T signal is called Analog signal. If a time signal can take on only a finite no. of distinct values then signal is called Digital signal.

③ Real / Complex Signals

A signal $x(t)$ is a real signal if its value is a real number & a signal $x(t)$ is complex signal if its value is a complex number. A general complex no. can be in form of

$$x(t) = x_1(t) + j\,x_2(t)$$

$$x_1(t) = \mathrm{Re}\{x(t)\}$$
$$x_2(t) = \mathrm{Im}\{x(t)\}$$

(4) Deterministic / Random Signal

Those signals that can be modeled or expressed mathematically

eg

$$x(t) = 3\cos(200\pi t + 30^\circ)$$

is a deterministic signal.

Random signal w/c can't expresed mathematically.

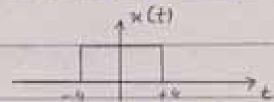
* Random signals are characterized statistically e.g noise, temp

(5) Even / Odd Signals

A signal $x(t)$ is referred to as an even fns if $x(t) = x(-t)$ {C-T}

or

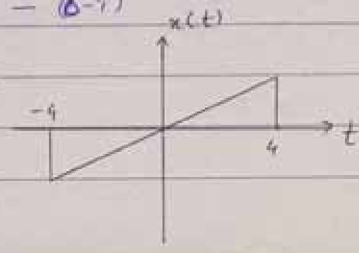
$x[n] = x[-n]$ {D-T}



A signal $x(t)$ is referred to as an Odd signal if $x(t) = -x(-t)$ → (C-T)

or

$x[n] = -x[-n]$ — (D-T)



(6) Periodic / A-periodic signals;

A fns that repeat itself after a regular interval is called Periodic Signal

(7) Energy / Power Signals:

A C-T or D-T signal is said to be an energy signal iff $0 < E < \infty$ and so $P = 0$.

A C-T or D-T signal is said to be a power signal iff $0 < P < \infty$ and so $E = \infty$.

where,

(C-T)
$$E = \lim_{L\to\infty} \int_{-L}^{L} |x(t)|^2\, dt$$

(D-T)
$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

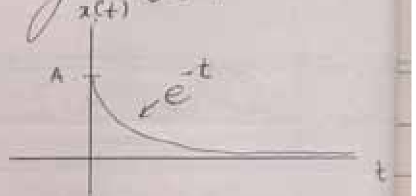
and

(C-T)
$$P = \lim_{L\to\infty} \frac{1}{2L} \int_{L}^{-L} |x(t)|^2\, dt$$

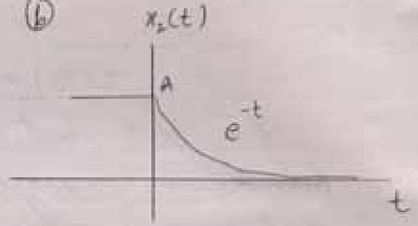
(D-T)
$$P = \frac{1}{2N+1} \sum_{n=-\infty}^{\infty} |x[n]|^2$$

Q1 => Determine whether following fns are energy or power signals -

Sol ⓐ for signal $x_1(t)$

ⓐ 

$$E = \lim_{L\to\infty} \int_0^\infty |(Ae^{-t})|^2 dt$$

ⓑ 

$$E = \lim_{L\to\infty} A^2 \int_0^\infty e^{-2t} dt$$

$$E = \lim_{L\to\infty} \frac{A^2}{-2} \left| e^{-2t} \right|_0^\infty$$

$$E = -\frac{A^2}{2} \{ e^{-\infty} - e^{0} \} \Rightarrow \frac{A^2}{2}$$

So, $0 < E < \infty$ is satisfied

Now

$$P = \lim_{L\to\infty} \frac{1}{2L} \int_0^\infty |Ae^{-t}|^2 dt$$

$$P = \lim_{L\to\infty} \frac{1}{2L} \left(\frac{A^2}{2} \right)$$

Applying limit.

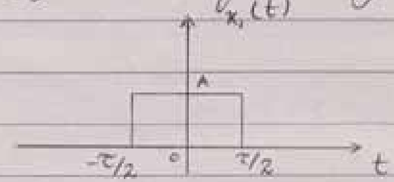
$$P = \frac{1}{\infty} \left(\frac{A^2}{2} \right)$$

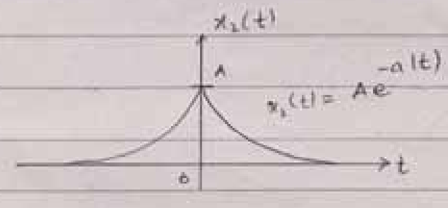
$P = 0$ Hence $0 < E < \infty$

$P = 0$

Thus signal is an Energy sig

Find energy of following signals :-

ⓐ 

ⓑ 

Q1 (b) for signal $x_2(t)$

$$E = \lim_{L\to\infty}\left[\int_{-\infty}^{0}|A|^2 dt + \int_{0}^{+\infty}|Ae^{-t}|^2 dt\right]$$

$$E = \lim_{L\to\infty}\left[A^2\int_{-\infty}^{0}(1)\,dt + A^2\int_{0}^{\infty}e^{-2t}dt\right]$$

This expression is boundless hence has infinite value.

$$E = \infty$$

$$P = \lim_{L\to\infty}\frac{1}{2L}\left[A^2\int_{-\infty}^{0}(1)\,dt + A^2\int_{0}^{\infty}e^{-2t}dt\right]$$

$$P = \lim_{L\to\infty}\frac{1}{2L}\left[A^2|0-(-\infty)| - \frac{A^2}{2}\left|e^{-2t}\right|_{0}^{\infty}\right]$$

$$P = \lim_{L\to\infty}\frac{1}{2L}\left[A^2(\infty) - \frac{A^2}{2}(-1)\right]$$

$$P = \frac{1}{2(\infty)}\left[A^2(\infty) + \frac{A^2}{2}\right]$$

$$P = \frac{A^2}{2}\left(\frac{\infty}{\infty}\right) + \frac{A^2}{4\infty}$$

$$P = \frac{A^2}{2}$$

Hence signal is Power Signal.

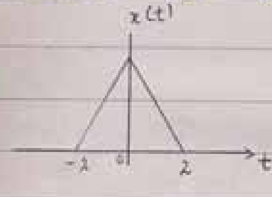
Basic Operations on Signals

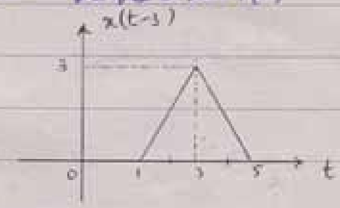
(or Transformation)

Aug 07

① The time-shifting operation

The signal $x(t-t_0)$ is time shifted version of signal $x(t)$ by amount t_0 in +ve direction.



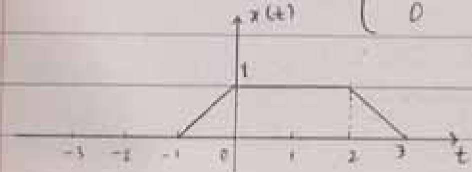


Physically 't_0' cannot take on -ve values but from analytical view point $x(t-t_0)$ for $t<0$ i.e $x(t+t_0)$ represents the advanced replica of $x(t)$

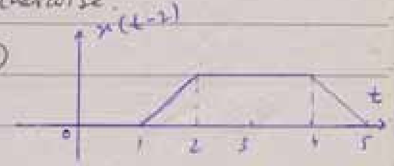
Q Write expression and draw graph of shifted signal? (a) $x(t-2)$ (b) $x(t+3)$

for

$$x(t) = \begin{cases} t+1 & -1\le t\le 0 \\ 1 & 0<t\le 2 \\ -t+3 & 2<t\le 3 \\ 0 & \text{otherwise} \end{cases}$$



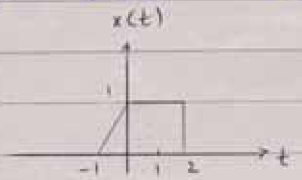
(a)

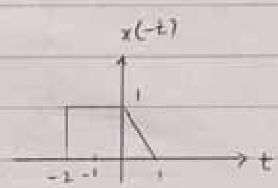


$$x(t-2) = \begin{cases} t-1 & 1\le t\le 2 \\ 1 & 2<t\le 4 \\ -t+5 & 4<t\le 5 \\ 0 & \text{otherwise} \end{cases}$$

② **Reflection Operation:**

The signal $x(-t)$ is obtained by reflection or mirror image along the axis $t=0$





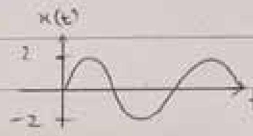
$$x(t) = \begin{cases} t+1 & -1 \le t \le 0 \\ 1 & 0 \le t \le 2 \\ 0 & \text{otherwise} \end{cases} \qquad x(-t) = \begin{cases} 1 & -2 \le t \le 0 \\ -t+1 & 0 < t \le 1 \\ 0 & \text{otherw.} \end{cases}$$

③ **Time-Scaling operation:**

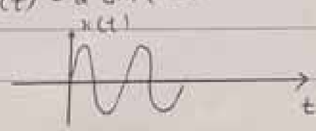
If the independant variable is scaled by a parameter "α" then $x(\alpha t)$ is a compressed version of $x(t)$ if α is greather than 1 $(\alpha > 1)$.

(The signal exist in a smaller time interval) and is an expanded version if $|\alpha| < 1$.

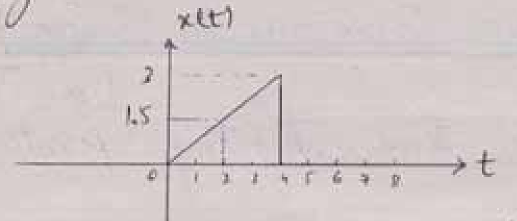
$x(t) = 2\sin(t)$



$x(t) = 2\sin(2t)$



Q A C-T signal $x(t)$ is shown in following figure:

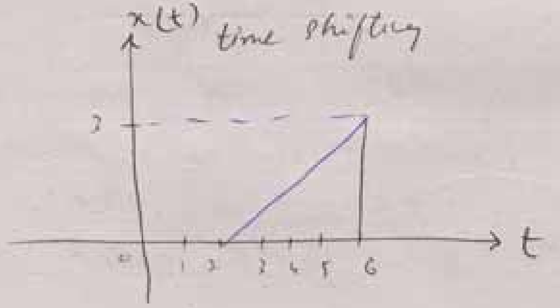


* Sketch and label each of following signal

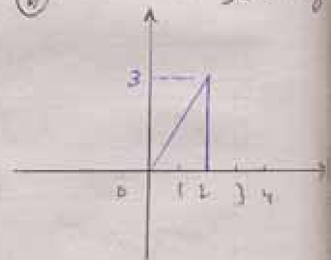
ⓐ $x(t-2)$ ⓑ $x(2t)$

ⓒ $x(-t)$

ⓐ time shifting

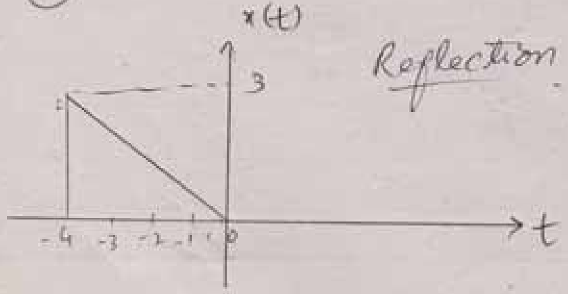


ⓑ Scaling



ⓒ

Reflection



Q2 For signal $x(t)$ has given find

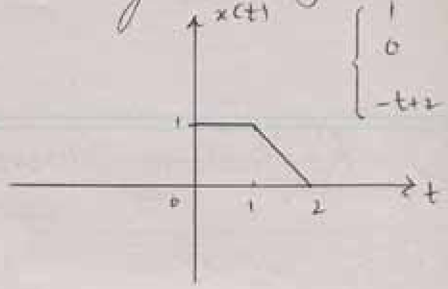
$\begin{cases} 1 & 0\le \\ 0 & 0t \\ -t+2 & 1\le \end{cases}$

(a) $x(t+1)$

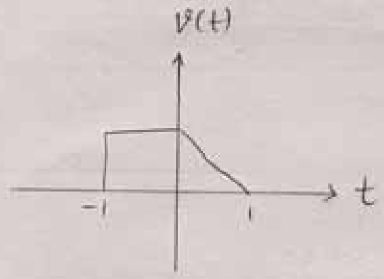
(b) $x(-t+1)$

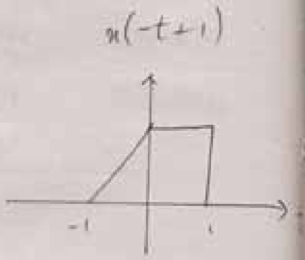
(c) $x(\frac{3}{2}t)$

(d) $x(\frac{3}{2}t+1)$

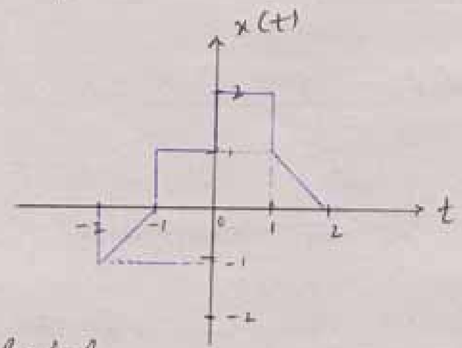


(b)





Q3 For given signal $x(t)$ of as shown



sketch & label

(a) $x(t-1)$ (b) $x(2-t)$

(c) $x(2t+1)$ (d) $x(4-t/2)$

(e) $2x(-t)$

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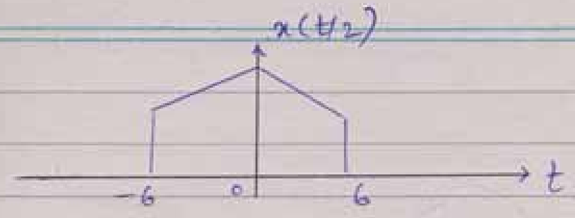


Signal Representation & Models

A signal can represented in two ways:

(i) Mathematically.

(ii) Graphically.

Both have advs to in different cases.

* eg It is easier to illustrate mathematically if signal is Periodic or analog in nature.

* Signal deffined piece-wise are better illustrated & understandin graphical method.

Complex Exponentials & Sinosoidal Signals

* The C-T C. exponential signal is of the form-

$$x(t) = ce^{at}$$

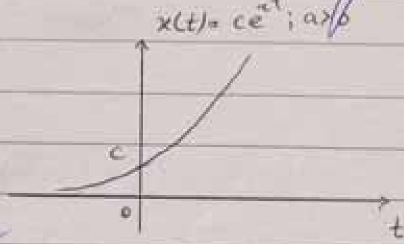
where 'c' & 'a' are both complex in general.

(A) Real Exponential Signal

When Both 'c' & 'a' are Real numbers the resulting exponential signal is referred to as Real-exponential signal.

(i) Growing Real exponential signal:

If 'a' is +ve then as 't' inc↑ fns x(t) also inc↑ non-linearly.

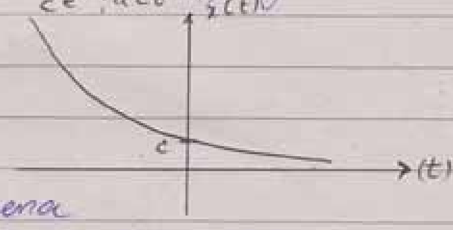


* It is a form i.e used in decribing many diff physicall procesess including chain reaction in atomic explosion & complex chemical reaction.

(Real Exponential signal)

(ii) Decaying Real-exponential Signal:

If 'a' is -ve; than, the exponential fns is reffered to as decaying exponential.



* It is a form i.e used to decribe a varity of phenomena including process of radio-active decay response of R.C ckts & damped mechanical system.

(B) Periodic Exponential & Sn. Signal

In this case 'c' is a real number, whereas 'a' is purely imaginary.

i.e $a = j\omega_0$

Now the exponential signal will beome

$$x(t) = ce^{j\omega_0 t}$$

Using Euler's identity

$$x(t) = c\,[\cos(\omega_0 t) + j\sin(\omega_0 t)] = ce^{j\omega_0 t}$$

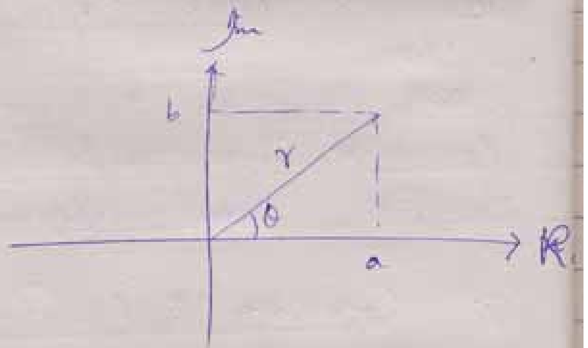
This is a periodic signal as Cosine & Sine signals are periodic.

Recall :-

$$x = a + jb$$

* It is called Rectangular or Cartesian form.

$$x = re^{j\theta}$$



$$x = r\cos\theta$$
$$y = r\sin\theta$$

$$e^{j\theta} = \cos\theta + j\sin\theta \qquad (r = 1)$$

(By Euler's Identity.)

07) The time period of these signals is

$$T = \frac{2\pi}{\omega_0}$$

* A signal closely related to periodic exponential signal is Sinusoidial signal i.e.

$$x(t) = C\cos(\omega_0 t + \phi)$$

* Periodic exponentials are useful to consider harmonic signals.

© General - Complex - exp - Signal

In this case

'a' & 'c' → Complex number.

Let us represent 'a' in rectangular form i.e

$$a = r + j\omega_0$$

And 'c' in polar form i.e

$$c = |c|\, e^{j\phi t}$$

then; The exponential signal becomes,

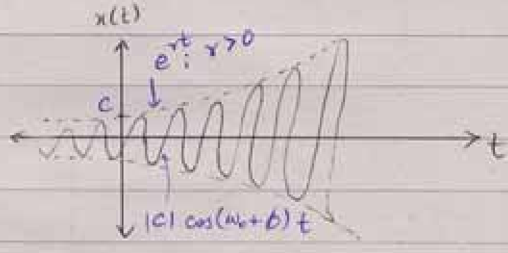
$$x(t) = |c|\, e^{j\phi t} \cdot e^{(r + j\omega_0)t}$$

...neral - comp-exp-signal?

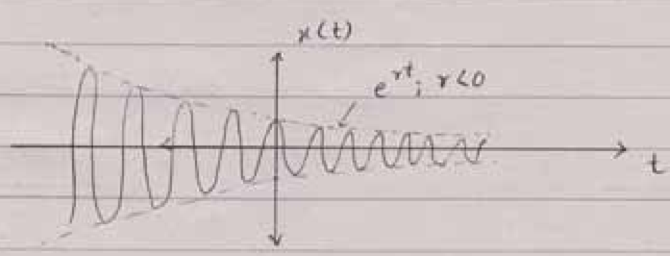
g 07)

$$x(t) = |c|\, e^{j\phi t} \cdot e^{(r + j\omega_0)t}$$
$$= |c|\, e^{j\phi t} \cdot e^{rt} \cdot e^{j\omega_0 t}$$
$$= |c|\, e^{rt} \cdot e^{j(\omega_0 + \phi)t}$$
$$x(t) = |c|\, e^{rt} \cdot \cos(\omega_0 + \phi)t$$

If $r > 0$



If $r < 0$



These two signals are called "damped sinusiods."
eg The responses of RLC circuits, mechanical systems containing both restoring and damping forces and complex harmonic analysis.

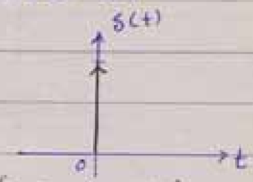
① **IMPULSE FUNCTION**

(UNIT - IMPULSE)

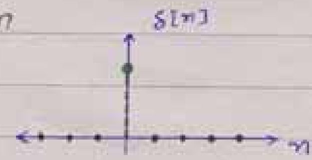
Aug 07)

Unit-impulse signal is defined as

$$\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & t \neq 0 \end{cases}$$

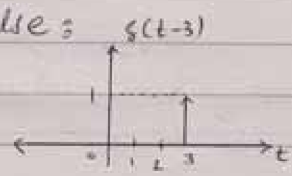


In D-T domain

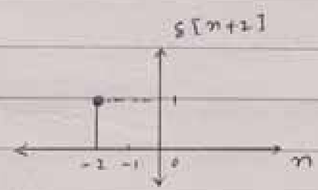


Shifted impulse:

$\delta(t-3)$



$\delta[n+2]$

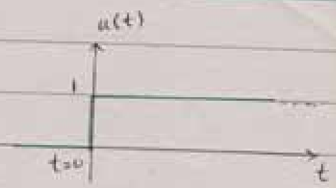


[Aug 07]

UNIT - STEP FUNCTION {u(t) or u[n]}

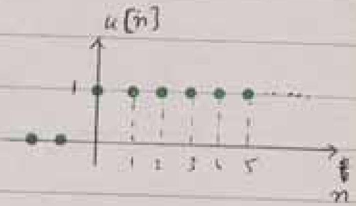
It is defined as

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$



and in discrete t-domain,

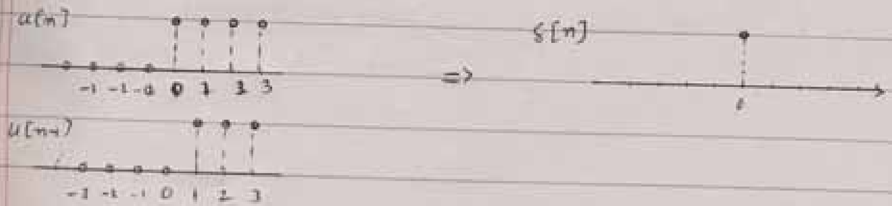
$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$



Relation-ship b/w 'δ' & 'u'

D-T domain:

① $\delta[n] = u[n] - u[n-1]$



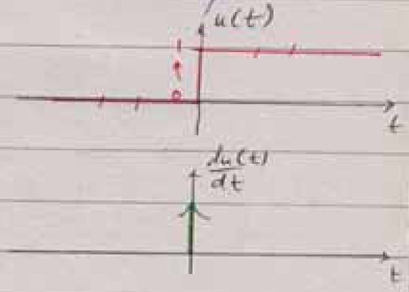
② $u[n] = \sum_{m=0}^{\infty} \delta[n-m]$

The D-T unit step signal is running sum of unit impulses.

[Aug 07] C-T domain:

③ $\delta(t) = \frac{du(t)}{dt}$

Continues Time unit impulse is 1st derivative of C-T unit step.



④ C-T unit step is running integral of unit impulses.

$$u(t) = \int_0^{\infty} \delta(t-\tau)\, d\tau$$

'τ' is arbitary variable. (Running Variable) of time 't'.

07) (Systems) C-T SYSTEMS

Types of Systems:

1. Linear / Non-linear system.
2. Time - Varient / Time - Invarient system.
3. Memory / Memory - less system.

Linear Time Invarient System (LTI)

→ Linear systems are those having response obeying linearity i.e.

If $x_1(t) \rightarrow y_1(t)$

$x_2(t) \rightarrow y_2(t)$

then

$$a_1 x_1(t) + a_2 x_2(t) \rightarrow a_1 y_1(t) + a_2 y_2(t)$$

→ Time invarient systems are those which in a time delay or in time advance of I/p signal leads to an identical time shift in O/p signal is called time invarient.

If $x(t) \rightarrow y(t)$

then $x(t-t_0) \rightarrow y(t-t_0)$

For,

Short forms:

System → Sym

Signal → Snl

Response of LTI Systems

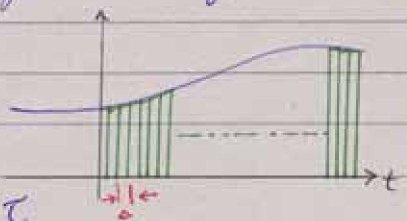
(A) Response on Unit-Impulse Signal (Impulse Response); {h(t)}

The h(t) of a C-T LTI system is defined to be response of system when i/p is $\delta(t)$.
i.e.

$$h(t) = \text{Response}\{\delta(t)\}$$

(B) Respone To an Arbitrary Snl {x(t)}

As any signal can be represented as a "continuous - Integral" of weighted impulses.



for $\Delta \to 0$

$$x(t) = \int_{-\infty}^{+\infty} \delta(t-\tau).\,x(\tau)\,d\tau$$

$x(\tau)$ → Amplitude at point 'τ'

Since Sym. is linear the response y(t) of Sym to an arbitrary i/p x(t) can expressed as;

$$y(t) = \text{Response}\{x(t)\} = \int_{-\infty}^{\infty} x(\tau).\,h(t-\tau)\,d\tau$$

$$y(t) = \int_{-\infty}^{\infty} h(t-\tau).\,x(\tau)\,d\tau$$

It indicates that a C-T LTI system is completely characterized by its impulse response h(t).

This equation also defines a mathematical operation b/w two C-T Snl called Convolution. denoted by "$*$"
i.e

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau).\,h(t-\tau)\,d\tau = y(t)$$

* It is generally called "Convolution-Integral"

Properties of LTI System.

1. Memoryless:
A sym is linear & memoryless (& also time invarient) if o/p depends only present value of i/p.
eg
$$y(t) = k\,x(t)$$

2. CAUSALITY; (Physically realizable)
It means o/p of system at 't_0' depends only on values of i/p for $t \le t_0$ (Only present & past values)

107) {Causality}

$h(t) = 0 \qquad t < 0$

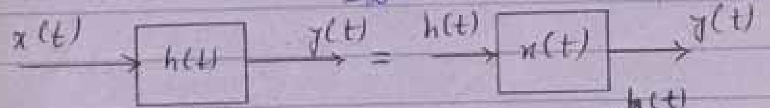
③ Commutativity : (Property of Convolution).

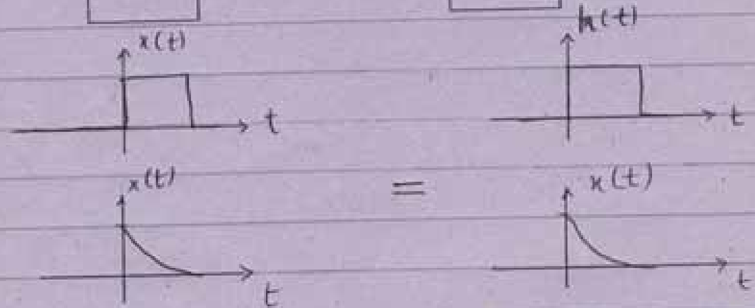
A sym would be commutative b/c convolution is a com-" - operation i.e.

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau).\, h(t-\tau) = y(t)$$

and

$$h(t) * x(t) = \int_{-\infty}^{\infty} x(t-\tau)\, h(\tau)\, d\tau = y(t)$$





④ Stability : (BIBO → Bounded i/p bounded o/p)

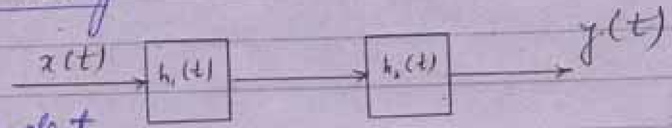
LTI is BIBO (stable) if its impulse response is absolutely integrable i.e

$$\int_{-\infty}^{\infty} |h(\tau)|\, d\tau < \infty$$

⑤ Associativity :

$$x(t) * h_1(t) * h_2(t) = [x(t) * h_1(t)] * h_2(t)$$
$$= x(t) * [h_1(t) * h_2(t)]$$

Graphically

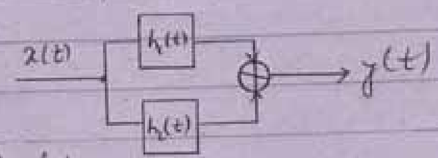


is equivalent

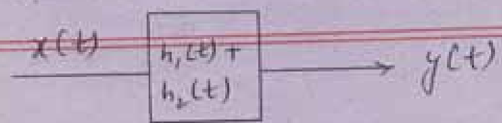
Associativity employees that a cascade combination of LTI sym can be replaced by snl sym whose impulse response is the convolution of individual impulse responses.

⑥ Distributivity : (Property of Convolution)

$$x(t) * h_1(t) + x(t) * h_2(t) = x(t) * [h_1(t) + h_2(t)]$$



is equivalent to



Distributivity:

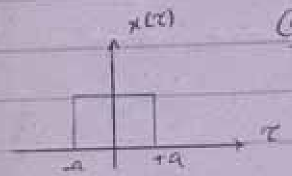
It states that a parallel combination of LTI system is equivalent to a single sym whose response is the sum of individual response impulse responses

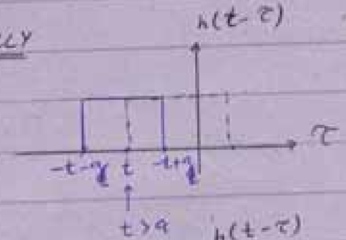
Ques Find o/p of an LTI system with i/p $x(t)$ & impulse response $h(t)$ as shown:

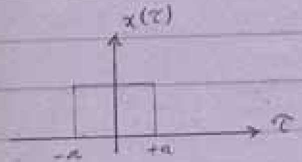
Convolution Equation: $y(t) = \int_{-\infty}^{\infty} x(\tau).\, h(t-\tau)\, d\tau$ $\quad h(\tau+t)$

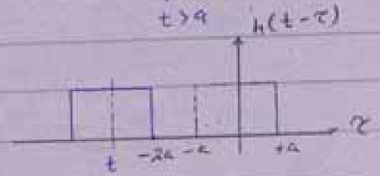
Solⁿ

GRAPHICALLY







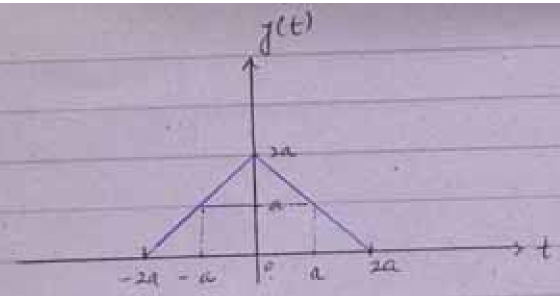


for $t < -2a$

$x(\tau).\, h(t-\tau) = 0 \rightarrow y(t) = 0$

$-2a \le t \le 0$

$x(\tau).\, h(t-\tau)$ is increasing



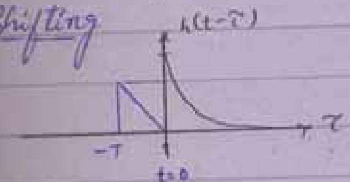
* In convolution process one signal is kept constant and other is shifted on it & the overlapping area is ploted (Calculated)

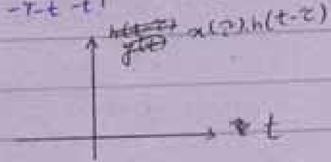
Prob Let $x(t) = Ae^{-t}.\, u(t)$ and $h(t) = \frac{t}{T}$ for $0 \le t < T$

Find output if system is LTI.

Solⁿ Analytically ← (Graphically 1st)

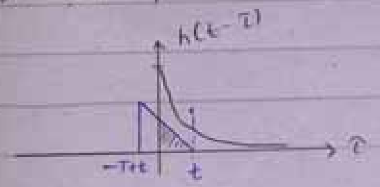
Shifting

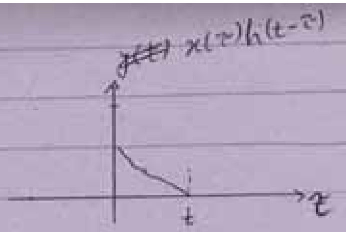




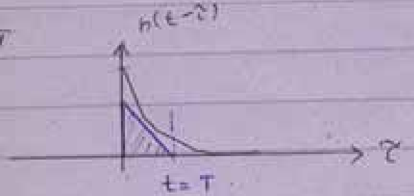
for $t = 0$

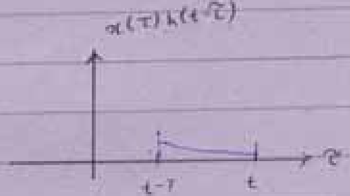
for $t > 0$, $t < T$



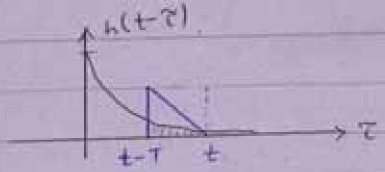


for $t = T$

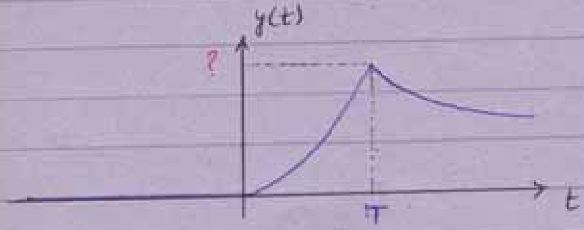




for $t > T$



Output



Analytically

$$y(t) = \int_{-\infty}^{\infty} x(\tau).\, h(T-\tau)\, d\tau \quad \text{—— ①}$$

$$y(t) = \int_{0}^{t} \quad \text{- - - - - -}$$

{Convolution}

Analytically

$$y(t) = \int_{-\infty}^{\infty} x(\tau).\, h(t-\tau)\, d\tau \quad \text{—— ①}$$

for $0 \to T$

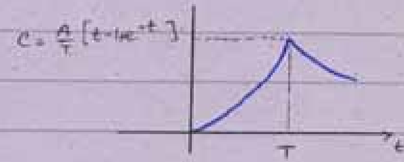
Hence

$x(\tau)$ $\quad h(t-\tau)$ $\quad$ As $h(t) = \frac{t}{T}$

$$y(t) = \int_{0}^{t} (Ae^{-\tau})\left(\frac{t-\tau}{T}\right) d\tau$$

$$y(t) = \frac{A}{T}\int_{0}^{t} e^{-\tau}(t-\tau)\, d\tau$$

$$y(t) = \frac{A}{T}\left[t - 1 + e^{-t}\right] \qquad 0 < t < T$$

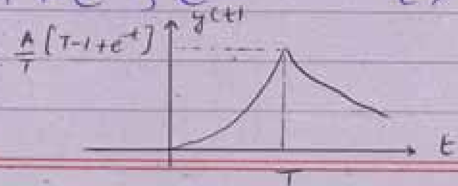


for $t > T$

$$y(t) = \int_{t-T}^{t} (Ae^{-\tau})\left(\frac{t-\tau}{T}\right) d\tau$$

$$= \frac{A}{T}\int_{t-T}^{t} e^{-\tau}(t-\tau)\, d\tau$$

$$y(t) = \frac{A}{T}\left[T - 1 + e^{-t}\right] e^{-(t-T)} \qquad t > T$$



Response of LTI System on Complex Sinusoidal Input

Let impulse response of any LTI system is $h(t)$ and input signal is

$$x(t) = e^{j\omega t}$$

Output $y(t)$ of LTI System is given by convolution integral is

$$y(t) = \int_{-\infty}^{\infty} x(\tau).\, h(t-\tau)\, d\tau$$

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, e^{j\omega(t-\tau)}\, d\tau$$

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, e^{j\omega t} . e^{-j\omega\tau}\, d\tau$$

$$y(t) = e^{j\omega t} \int_{-\infty}^{\infty} h(\tau).\, e^{-j\omega\tau}\, d\tau$$

$$y(t) = x(t) \int_{-\infty}^{\infty} h(\tau).\, e^{-j\omega\tau}\, d\tau$$

Can be written as

$$\boxed{y(t) = x.(t)\ H(j\omega)} \quad \text{——— (1)}$$

where $H(j\omega)$ is not a fns of time 't' but only fns of frequency 'ω'.

...med as frequency response of CT LTI System.

Hence,

$$H(j\omega) = \int_{-\infty}^{\infty} h(\tau).\, e^{-j\omega\tau}\, d\tau$$

As $H(j\omega)$ is a complex number.

$$H(j\omega) = |H(j\omega)|.\, e^{\arg\{H(j\omega)\}}$$

where

$|H(j\omega)|$ is "Amplitude Response" (Spectrum)

and $\arg\{H(j\omega)\}$ is called "Phase-Response".

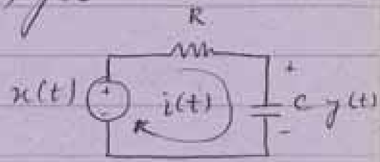
eq (1) gives,

$$y(t) = e^{j\omega t}.\left\{ |H(j\omega)|\, e^{\arg\{H(j\omega)\}} \right\}$$

$$y(t) = |H(j\omega)|\ e^{j\{\omega t + \arg\{H(j\omega)\}\}}$$

* It state that System modifies the amplitude of input by modulus of $|H(j\omega)|$ and phase by $\arg\{H(j\omega)\}$

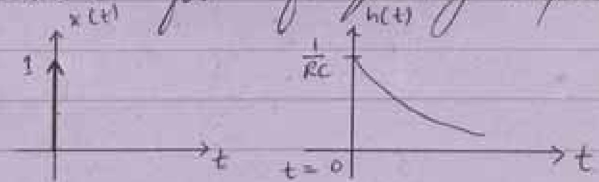
Q.No.1 Impulse response of system



* Relating the input voltage $x(t)$, to output voltage across capacitor $y(t)$ is given by:

$$h(t) = \frac{1}{RC} e^{-t/RC} \cdot u(t)$$

Find expression for frequency response.



we have

$$H(j\omega) = \int_{-\infty}^{\infty} h(\tau)\, e^{-j\omega\tau}\, d\tau$$

$$= \int_{0}^{\infty} \frac{1}{RC} e^{-\tau/RC} \cdot e^{-j\omega\tau}\, d\tau$$

$$= \frac{1}{RC} \int_{0}^{\infty} e^{-\left(\frac{1}{RC}+j\omega\right)\tau}\, d\tau$$

$$= \frac{1}{RC} \cdot \frac{1}{\left(-\frac{1}{RC}-j\omega\right)} \left| e^{-\left(\frac{1}{RC}+j\omega\right)\tau} \right|_{0}^{\infty}$$

$$H(j\omega) = \left(\frac{1}{RC}\right)\left(\frac{-1}{\frac{1}{RC}+j\omega}\right)\left\{0-1\right\}$$

$\downarrow$
a

$$H(j\omega) = \frac{1/RC}{\left(\frac{1}{RC}+j\omega\right)}$$

$$\left|\frac{1}{1+j\ldots}\right.$$

$$|H(j\omega)| = \frac{1/RC}{\sqrt{\left(\frac{1}{RC}\right)^2 + (\omega)^2}} \longrightarrow \text{Amplitude Response}$$

$$\arg\{H(j\omega)\} = \tan^{-1}(\omega RC) \longrightarrow \text{Phase Response}$$

Q.No.1

(..08.07)

Eigen Function

Recall that for $x(t) = e^{+j\omega t}$

the LTI system given o/p $y(t) = e^{j\omega t} . H(j\omega)$

where frequency response

$H(j\omega)$ is defined as

$$H(j\omega) = \int_{-\infty}^{\infty} h(\tau) e^{-j\omega\tau} d\tau$$

We say that any complex sinusoid $\psi(t) = e^{j\omega t}$ is an Eigen function of system.

..g 07

Fourier Analysis

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega t}$$

① Fourier Series

By analysis of Fourier any periodic signal $x(t)$ can be represented as a series of complex exponentials.

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \quad \text{--- ①}$$

where

ω_0 = Time period for the waveform repeats itself.

or

$$\omega_0 = \frac{2\pi}{T_0} \quad (\text{rad/sec})$$

'T_0' is time-period.

And

'a_k' is $a_k = \frac{1}{T} \int_T x(t) . e^{-j\omega_0 k t} dt$

The set of co-efficients are often called "Fourier-Series co-efficients or Spectral coefficients of $x(t)$".

{Fourier Series}

For $k=0$, Zero Frequency Component.

$$a_{k=0} = a_0 = \frac{1}{T}\int_T x(t)\,dt$$

(Average Value)

It is called 'DC' or constant component of $x(t)$ w/c is simply average value of $x(t)$ over One Period.

Using Euler's Identity, i.e

$$e^{jk\omega t} = \cos k\omega t + j \sin k\omega t$$

the eq ① can be written as;

$$x(t) = a_0 + \sum_{k=1}^{\infty}\left(a_k \cos k\omega_0 t + b_k \sin k\omega t\right) \quad ②$$

$a_0 \rightarrow$ DC Value.

$a_k \rightarrow$ Fourier Coefficients

$b_k \rightarrow$ - " - " -

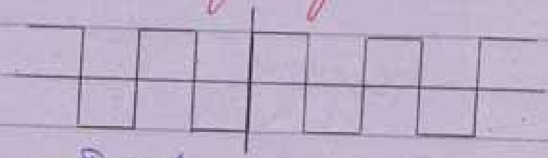
(08,07) where

$$a_0 = \frac{1}{T}\int_T x(t)\,dt$$

$$a_k = \frac{2}{T}\int_T x(t) \cos k\omega_0 t\,dt$$

$$b_k = \frac{2}{T}\int_T x(t) \sin k\omega_0 t\,dt$$

Q_1 Find Fourier Series of Signal $x(t)$ as shown



$T_0 = 4$ sec

$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{4} = \frac{\pi}{2}\ \text{rad/sec}$$

$$a_0 = \frac{1}{4}\left[\int_0^2 (A)\,dt + \int_2^4 (-A)\,dt\right]$$

$$= \frac{A}{4}\left(\int_0^2 dt - \int_2^4 dt\right)$$

$$= \frac{A}{4}\left[\,|t|_0^2 - |t|_2^4\,\right]$$

$$= \frac{A}{4}\left[2 - 2\right] \qquad \boxed{a_0 = 0}$$

Q1

Now find a_k:

$$a_k = \frac{1}{T}\int_T x(t).\cos k\omega_0 t\,dt.$$

$$= \frac{1}{4}\left[\int_0^2 (A)\cos k\frac{\pi}{2}t\,dt + \int_2^4 (-A)\cos k\frac{\pi}{2}t\,dt\right]$$

$$= \frac{A}{4}\left[\int_0^2 \cos k\frac{\pi}{2}t\,dt - \int_2^4 \cos k\frac{\pi}{2}t\,dt\right]$$

$$a_k = \frac{A}{4}\left[\frac{2}{k\pi}\left|\sin k\frac{\pi}{2}t\right|_0^2 - \frac{2}{k\pi}\left|\sin k\frac{\pi}{2}t\right|_2^4\right]$$

$$a_k = \frac{A}{2k\pi}\left[\{\sin k\pi - \sin 0\} - \{\sin 2k\pi - \sin k\pi\}\right]$$

$$a_k = \frac{A}{2\pi k}\left[(0-0)-(0-0)\right] \text{ for all "k"}$$

$$\boxed{a_k = 0}$$

Now finding b_k:

$$b_k = \frac{2}{T}\int_T x(t).\sin k\omega_0 t\,dt$$

$$b_k = \frac{2}{4}\left[\int_0^2 (A)\sin k\frac{\pi}{2}t\,dt + \int_2^4 (-A)\sin k\frac{\pi}{2}t\,dt\right]$$

Q1

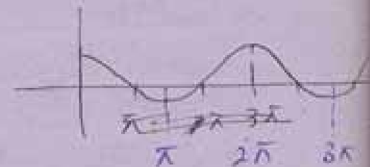
$$b_k = \frac{A(2)}{4}\left[\int_0^2 \sin k\frac{\pi}{2}t\,dt - \int_2^4 \sin k\frac{\pi}{2}t\,dt\right]$$

$$b_k = \frac{-A(2)}{4}\left[\frac{2}{k}\right]\left[\left|\cos k\frac{\pi}{2}t\right|_0^2 - \left|\cos k\frac{\pi}{2}t\right|_2^4\right]$$

$$= \frac{-A(2)}{2k\pi}\left(\{\cos k\pi - \cos 0\} - \{\cos 2k\pi - \cos\right.$$

$$b_k = \frac{-A}{k\pi}\left[\{\cos k\pi - \cos 0\} - \{\cos 2k\pi - \cos k\pi\}\right]$$

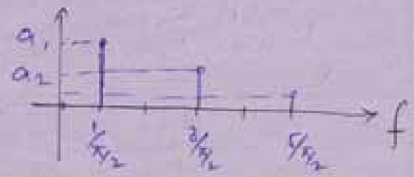
for even "k"



$$b_k = 0$$

$$x(t) = \frac{4A}{\pi}\sum_{\substack{k=1\\k=odd}}^{\infty}\left(\frac{1}{k}\cdot\sin k\right.$$

for Odd "k"



$$b_k = \frac{4A}{k\pi}$$

we have

$$x(t) = a_0 + \sum_{k=1}^{\infty} a_k\cos k\omega_0 t + \sum_{k=1}^{\infty} b_k\sin$$

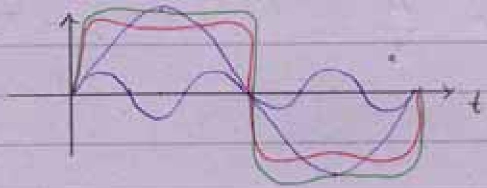
$$x(t) = 0 + 0 + \sum_{k=1}^{\infty}\left(\frac{4A}{k\pi}\right)\sin k\omega_0$$

$$x(t) = \sum_{\substack{k=1\\k=odd}}^{\infty}\left(\frac{4A}{k\pi}\right)\sin k\omega_0 t$$

$$x(t) =$$

The frequency equivalent graph of signal $x(t)$ represented as $X(f)$ is called Frequency Response.

Q1 $$x(t) = \frac{4A}{\pi} \cdot \sum_{k=odd}^{\infty} \left(\frac{1}{k} \cdot \sin k \frac{\pi}{2} t \right)$$



Symmetry of Waveform

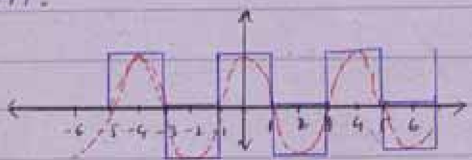
① Even Symmetry:

By mathematical definition:

If $x(t) = x(-t)$

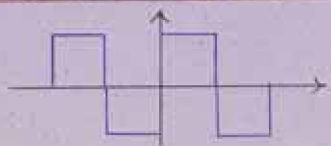
then

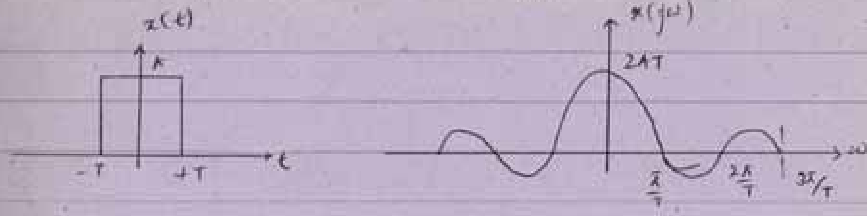
$x(t)$ is an Even Symmetric function. $\boxed{b_k = 0}$

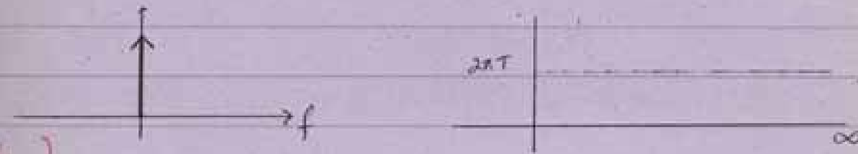


② Odd Symmetry:

$x(t) = -x(-t)$ $\boxed{a_k = 0}$







(1st Hourly)

Properties of Fourier Transform

① Linearity:

If $x(t) \xleftrightarrow{F.T} X(j\omega)$

and $y(t) \xleftrightarrow{F.T} Y(j\omega)$

then,

$$A\,x(t) + B\,y(t) \xleftrightarrow{F.T} A\,X(j\omega) + B\,Y(j\omega)$$

Proof: We have

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} dt$$

put $A\,x(t) + B\,y(t)$ inplace of "$x(t)$"

$$Z(j\omega) = \int_{-\infty}^{\infty} \left(A\,x(t) + B\,y(t) \right) e^{-j\omega t} dt$$

$$Z(j\omega) = \int_{-\infty}^{\infty} A\,x(t)\,e^{-j\omega t}\,dt + \int_{-\infty}^{\infty} B\,y(t)\,e^{-j\omega t}\,dt.$$

$$Z(j\omega) = A\int_{-\infty}^{\infty} x(t)\,e^{-j\omega t}\,dt + B\int_{-\infty}^{\infty} y(t)\,e^{-j\omega t}\,dt.$$

$$Z(j\omega) = F\{A\,x(t) + B\,y(t)\} \Rightarrow A\,X(j\omega) + B\,Y(j\omega)$$

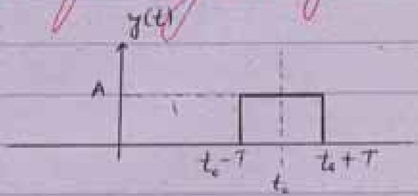
Hence proof of "Linearity".

② Time Shifting:

If $x(t) \longleftrightarrow X(j\omega)$

$x(t-t_0) \longleftrightarrow X(j\omega).\,e^{-j\omega t_0}$

Q Find fourier transform for signal $y(t)$ as shown:



we have

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\,e^{-j\omega t}\,dt$$

then

$$Y(j\omega) = \int_{t_0-T}^{t_0+T} (A)\,e^{-j\omega t}\cdot dt$$

$$= A\int_{t_0-T}^{t_0+T} e^{-j\omega t}\,dt$$

$$\Rightarrow \frac{-A}{j\omega}\left[e^{-j\omega t}\right|_{t_0-T}^{t_0+T}$$

$$\Rightarrow \frac{-A}{j\omega}\left[e^{-j\omega(t_0+T)} - e^{-j\omega(t_0-T)}\right]$$

$$\Rightarrow \frac{-A}{j\omega}\left[e^{-j\omega t_0}\cdot e^{-j\omega T} - e^{-j\omega t_0}\cdot e^{j\omega T}\right]$$

$$\Rightarrow \frac{A}{j\omega}\left[e^{j\omega T} - e^{-j\omega T}\right]e^{-j\omega t_0} \times \frac{2}{2}$$

$$\Rightarrow \frac{2A}{\omega}\left[\frac{e^{j\omega T} - e^{-j\omega T}}{2j}\right]e^{-j\omega t_0}$$

$$\Rightarrow \frac{2A}{\omega}\,\sin(\omega T) \times \frac{T}{T} \times e^{-j\omega t_0}$$

$$\Rightarrow 2AT\,.\,\frac{\sin(\omega T)}{\omega T}\,.\,e^{-j\omega t_0}$$

$$\Rightarrow 2AT\,\mathrm{sinc}(\omega T).\,e^{-j\omega t_0}$$

(Page 302 ; Oppenheim)

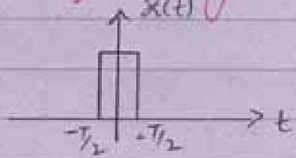
3407 {fourier Transform prop}

③ Time Scaling : (Page 308 Oppenheim)

If $x(t) \xleftrightarrow{F.T} X(j\omega)$

$x(\alpha t) \xleftrightarrow{F.T} \frac{1}{|\alpha|} X\left(\frac{j\omega}{\alpha}\right)$

Q1. Find fourier transform for signal $Z(t)$;



Sol Here

$Z(t) = x(2t)$

Now

$$Z(j\omega) = \int_{-T/2}^{+T/2} (A)\, e^{-j\omega t}\, dt.$$

$$= (A) \int_{-T/2}^{T/2} e^{-j\omega t}\, dt.$$

$$= \frac{-A}{j\omega}\left[e^{-j\omega T/2} - e^{+j\omega T/2}\right] \times \frac{2}{2}$$

$$= \frac{2A}{\omega}\left[\frac{e^{-j\omega T/2} - e^{j\omega T/2}}{2j}\right] \times \frac{T/2}{T/2}$$

$$= \frac{2AT}{2}\left[\frac{\sin(\omega T/2)}{\omega(T/2)}\right]$$

$$= \frac{2AT}{2}\left[\operatorname{sinc}\left(\frac{\omega T}{2}\right)\right]$$

3407 {Properties of F.T}

④ Convolution Property :

If $x(t) \xleftrightarrow{F.T} X(j\omega)$

$h(t) \xleftrightarrow{F.T} H(j\omega)$

then

$y(t) = x(t) * h(t) \xleftrightarrow{F.T} X(j\omega).H(j\omega) = Y(j\omega)$

Time-domain (Convolution) $\xleftrightarrow{F.T}$ f-domain (Multiplication)

Q Find the o/p of LTI System having impulse response $h(t) = \frac{\sin(2\pi t)}{\pi t}$ for an i/p $x(t) = \frac{\sin(\pi t)}{\pi t}$ ......?

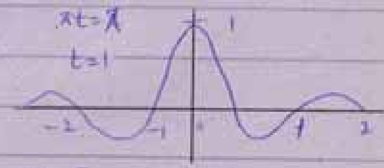
⑤ Duality :

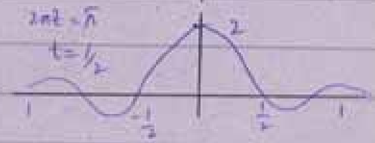
If $x(t) \longleftrightarrow X(j\omega)$

$X(t) \longleftrightarrow x(j\omega)$

Sol

$y(t) = x(t) * h(t)$

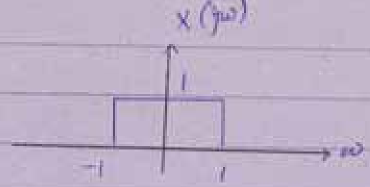




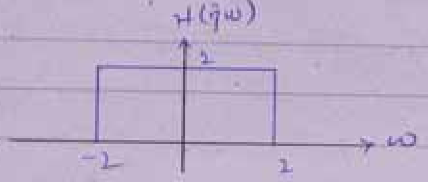
Using duality Property :

Q.

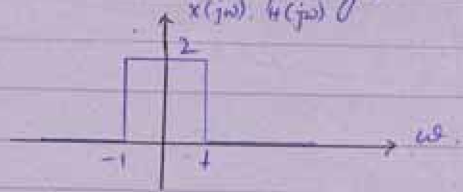
By duality:



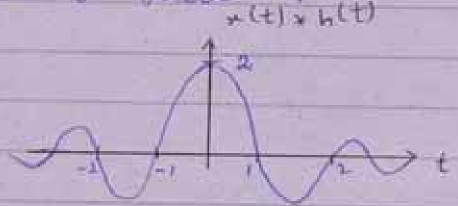
And



Using Convolution Property,



Again using duality property, we conclude that



$$y(t) = 2\,\frac{\sin \pi t}{\pi t}$$

Discrete – Time Signal

Defn:

① As name implies the discrete time signals are signals that are defined at discrete instants of time.

e.g.

① Temprature of patient after each hour.

② Stock exchange indexs at closing of each day.

③ Population of a country as obtained by servies.

② A second type of discrete time signal occur at when an analog C-T is converted into discrete-time signal by process of Sampling.

Representation of D-T Signals

In either case we represent D-T signal as a sequence of values. $x(t_n)$ where 't_n' correspond to instant at w/c signal is defined. Or equivalently as $x[n]$, where 'n' assuming only integer values.

09-07 [Representation of DT Signal]

① We usually represent D-T signal in functional form

eg

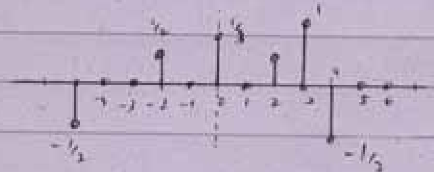
$x[n] = 2\cos 3\pi n$ *Where no finite interval defined.

or

$x[n] = 3n^2$

② If a signal is <u>non-zero</u> only over a finite interval than we list value of signal as element of a signal.

eg for $x[n]$



We can write sequence of $x[n]$ as

$$x[n] = \left\{ \frac{-1}{2}, 0, \frac{1}{2}, 0, \underset{\uparrow}{\frac{1}{3}}, 0, \frac{1}{2}, 1, -\frac{1}{2} \right\}$$

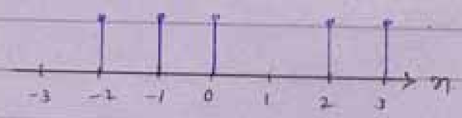
→ Arrow below member of a sequence indicates that this member exist at $n = 0$

09-07 **TRANSFORMATIONS**

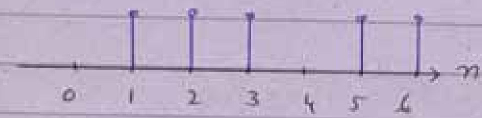
① Time - Shifting.
② Reflection.
③ Time - Scalling.

① **Time - Shifting :-**

for $x[n]$



$x[n-3]$ $n = 3$

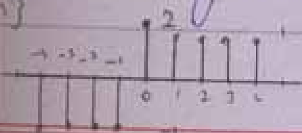


for $x[n] = \{0, 0, 1, \underset{\uparrow}{1}, 0, 1, 1\}$

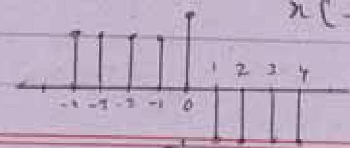
$x[n+2] = \{0, 0, 1, 1, 0, \underset{\uparrow}{1}, 1\}$

② **Reflection :-** $x[-n]$ represent the MIRRORED version of $x[n]$ about origin.

$x[n]$



$x[-n]$



$x[n] = \{-1, -1, -1, -1, \underset{\uparrow}{2}, 1, 1, 1, 1\}$

$x[-n] = \{1, 1, 1, 1, \underset{\uparrow}{2}, -1, -1, -1, -1\}$

(2.02) {Transformation D-T}

(i) **TIME SCALLING :-**

$x[\alpha n]$ represent Time-Scaled version of $x[n]$

* Time scalling must be interprated with care in D-T case since the sgls are defined only for integer value of 't' variable.

Q(n) $x[n] = e^{-n/2}\, u[n]$

Find

(a) $y[n] = x[2n]$

(b) $z[n] = x\left[\frac{5n}{3}\right]$

(a) Put $n=0$, $y[0] = x[2\times 0] \Rightarrow x[0] = 1$

$n=1$, $y[1] = x[2\times 1] \Rightarrow x[\cancel{0.5}\ 1] = e^{-1}$

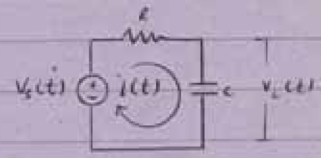
$n=2$, $y[2] = x[4] \Rightarrow e^{-2}$

$\vdots \quad \vdots \quad \vdots \quad \vdots$

'n' $\quad y[n] = x[2n] = e^{-n}$

(2.09) LTI → **SYSTEMS DESCRIBED BY DIFFERENTIAL EQs**

Applying KVL ;



$$V_s(t) = i(t)R + V_c(t) \quad \text{—} ①$$

We have,

$$i(t) = C\frac{dV_c}{dt}$$

$$① \quad V_s(t) = RC\frac{dV_c}{dt} + V_c(t) \quad \text{—} ②$$

Let $\quad V_s(t) = x(t)$, $V_c(t) = y(t)$

$$x(t) = RC\frac{dy(t)}{dt} + y(t)$$

$$\frac{1}{RC}x(t) = \frac{dy(t)}{dt} + \frac{1}{RC}y(t) \quad \text{—} (A)$$

(a) Constant Co-efficient 1st Order diff equation.

$$(A) \Rightarrow \quad \frac{dy(t)}{dt} + \frac{1}{RC}y(t) = \frac{1}{RC}x(t)$$

$$y' + \frac{1}{RC}y(t) = \frac{1}{RC}x(t)$$

* I/p - O/p relationship of RC circuit is described by the 1st order linear differential equation.

Similarly a system may contain high terms (diff order), hence general form of

N^{th} order linear constant co-efficient diff equation is given by;

$$\sum_{k=0}^{N} a_k \frac{d^k}{dt^k} y(t) = \sum_{k=0}^{M} b_k \frac{d^k}{dt^k} x(t)$$

* Examples of Systems:

$$y'(t) + 2y(t) = 3x(t) \quad \text{—— (A)}$$

$$2y'(t) - 4y(t) = 5x(t) \quad \text{—— (B)}$$

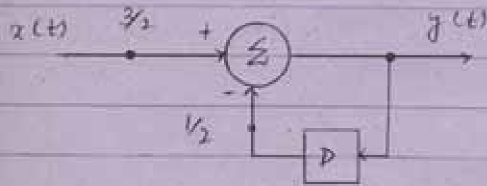
$$y''(t) = 4y'(t) - y(t) + 4x'(t) + 2x(t) \quad \text{—— (C)}$$

The System Block Diagram:

eq (a)

$$y'(t) + 2y(t) = 3x(t)$$

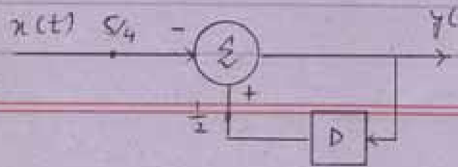
$$y(t) = \frac{3}{2}x(t) - \frac{1}{2}y'(t)$$



eq (B)

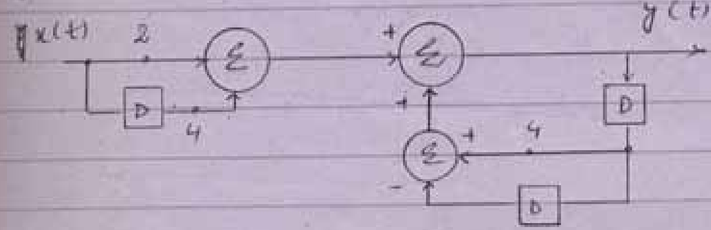
$$2y'(t) - 4y(t) = 5x(t)$$

$$y(t) = \frac{2}{4}y'(t) - \frac{5}{4}x(t)$$



eq (C) ⇒ $y''(t) = 4y'(t) - y(t) + 4x'(t) + 2x(t)$

$$y(t) = -y''(t) + 4y'(t) + 4x'(t) + 2x(t)$$



System Described by "Difference" EQ.

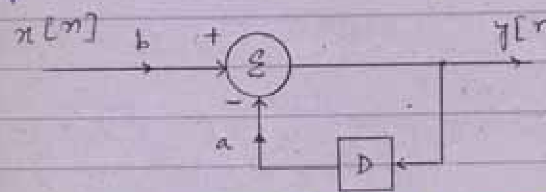
D-T system may also be described in difference equation model.

e.g.

$$y[n] + a\,y[n-1] = b\,x[n] \quad \text{—— (A)}$$

* where $y[n-1]$ represents a delayed sample of $y[n]$.

(A) ⇒ $y[n] = b\,x[n] - a\,y[n-1]$



Here "D" is a delay element.

* If a system is described in diff eq then system is known as 'Recursive' System.

(x-0?) General form of N^{th} order difference Eq:

$$\sum_{k=0}^{N} a_k\, y[n-k] = \sum_{k=0}^{M} b_k\, x[n-k]$$

In both cases of C-T & D-T differential & difference equation model, we note that the output is recursively appeared in the description that is why such system representation is reffered to as Recursive Systems.

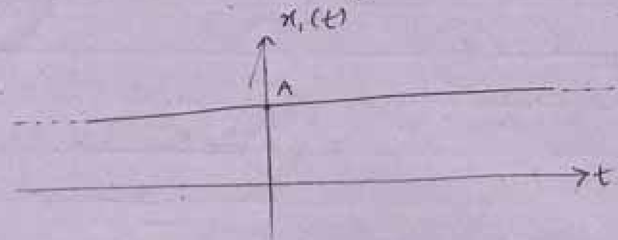
For, Short forms:

system → Sym

Signal → Snl

Backup example for new topic:

Consider a signal ⇒ $x_1(t) = A$



Taking fourier

$$X_1(j\omega) = \int_{-\infty}^{\infty} A e^{-j\omega t}\, dt$$

$$= \frac{A}{-j\omega} \left| e^{-j\omega t} \right|_{-\infty}^{\infty}$$

$$= \frac{-A}{j\omega} \left| \underset{0}{e^{-\infty}} - e^{+\infty} \right|$$

$$X_1(j\omega) = \infty$$

The fourier trans of $x_1(t) =$ doesn't exist (The integral doesn't converge

Short forms:
Sym
Syl

Recursive Systems.

LAPLACE TRANSFORMATION

$$F\{x(t).e^{\delta(t)}\} = \int_{-\infty}^{\infty} x(t)\, e^{-(\delta+j\omega)t}\, dt$$

put $s = \delta + j\omega$

$$= \int_{-\infty}^{\infty} x(t).e^{-st}\, dt.$$

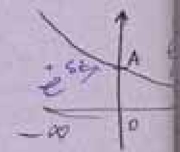
$$F\{x(t).e^{\delta(t)}\} = \mathcal{L}\{x(t)\}$$

* Fourier tranf is special case of Laplace tranf where $\delta = 0$.

Consider $x_2(t) = A\, e^{-\delta|t|}$ where $\delta \in$ Real

The fourier Transform for $x_2(t)$

$$X_2(j\omega) = \int_{-\infty}^{\infty} A\, e^{-\delta|t|}.\, e^{-j\omega}\, dt$$



$$X_2(j\omega) = A\int_{-\infty}^{\infty} e^{-\delta|t|}\, e^{-j\omega}\, dt$$

$$= A\left[\int_{-\infty}^{0} e^{+\delta t}.e^{-j\omega t}\, dt + \int_{0}^{\infty} e^{-\delta t}.e^{-j\omega t}\, dt\right]$$

$$= A\left[\int_{-\infty}^{0} e^{(\delta-j\omega)t}\, dt + \int_{0}^{\infty} e^{-(\delta+j\omega)t}\, dt\right]$$

$$X_2(j\omega) = A\,\frac{2\delta}{\omega^2+\delta^2} < \infty \quad \{\text{Converge}$$

It means that.

$$F\{x(t).e^{\delta(t)}\} = \int_{-\infty}^{\infty} x(t).e^{-j\omega t}.e^{-\delta(t)}\, d$$

Back to Topic.

Laplace Transform:

for a time-domain signal $x(t)$

the Laplace transform is

$$\mathcal{L}\{x(t)\} = X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

where

$$s = \sigma + j\omega$$

Ex: Consider signal $x(t) = e^{-at}\, u(t)$

find laplace trnf of $x(t)$?

Sol:

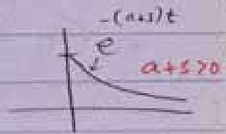
We have

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

$u(t)$ given

$$X(s) = \int_{0}^{\infty} e^{-at} \cdot e^{-st}\, dt$$

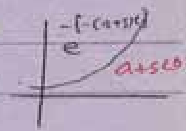
$$= \int_{0}^{\infty} e^{-(a+s)t}\, dt$$



only converges only $a+s>0 \rightarrow a + Re\{s\} > 0$

$\therefore Re\{s\} > -a$

$$X(s) = \frac{-1}{a+s} \left| e^{-(a+s)t} \right|_0^{\infty}$$



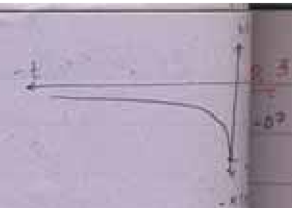
$$X(s) = \frac{-1}{a+s} \left| e^{-\infty} - e^{-0} \right|$$

$$X(s) = \frac{1}{s+a} \qquad \text{ROC}$$

05-10-07

Q2 ⇒ For signal $x(t) = -e^{-at} \cdot u(-t)$

Find $\mathcal{L}\{x(t)\}$ and ROC.



Sol

$$\mathcal{L}\{x(t)\} = X(s) = \int_{-\infty}^{\infty} x(t) \cdot e^{-st}\, dt$$

$$X(s) = \int_{-\infty}^{0} -e^{-at} \cdot e^{-st}\, dt$$

$$= -\int_{-\infty}^{0} e^{-(s+a)t}\, dt$$

$$= -\frac{(-1)}{s+a} \left[e^{-(s+a)t} \right]_{-\infty}^{0}$$

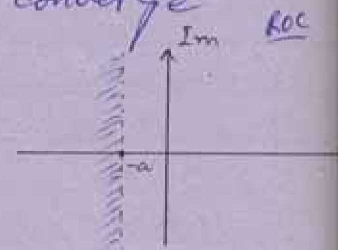
$$= \frac{1}{s+a} \left[e^{0} - e^{-(-\infty)} \right]$$

For this integral to converge

$(s+a) < 0$

$s < -a$

$Re\{s\} < -a$



$$X(s) = \frac{1}{s+a} \quad ; \quad \text{ROC}: Re\{s\} < -a$$

Find $\mathcal{L}\{x(t)\}$ and ROC for

$x(t) = 3e^{-2t}.u(t) - 2e^{-t}.u(t)$

Sol

$$\mathcal{L}\{x(t)\} = X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

$$\Rightarrow 3\int_0^{\infty} e^{-2t}. e^{-st}\, dt - 2\int_0^{\infty} e^{-t}. e^{-st}\, dt$$

$$\Rightarrow 3\int_0^{\infty} e^{-(s+2)t}\, dt - 2\int_0^{\infty} e^{-(s+1)t}\, dt$$

$$= 3. \frac{-1}{s+2} \left| e^{-(s+2)t} \right|_0^{\infty} - 2\frac{(-1)}{s+1} \left| e^{-(s+1)t} \right|_0^{\infty}$$

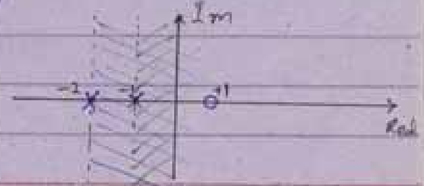
$$= \frac{-3}{s+2}\left[e^{-\infty} - e^{0}\right] + \frac{2}{s+1}\left[e^{-\infty} - e^{0}\right]$$

$$= \frac{-3}{s+2}\left[0 - 1\right] + \frac{2}{s+1}\left[0 - 1\right]$$

$$X(s) = \frac{3}{s+2} - \frac{2}{s+1} \quad \text{—— (A)}$$

for eq (A)

ROC: $s > -2$; ROC: $s > -1$

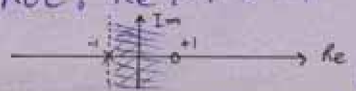


$$X(s) = \frac{3(s+1) - 2(s+2)}{(s+2)(s+1)}$$

$$= \frac{3s+3-2s-4}{(s+2)(s+1)}$$

$$X(s) = \frac{s-1}{(s+2)(s+1)}$$

ROC: $Re\{s\} > -1$



Ans

Region Of Convergance

ROC of a laplace Transform is defined "The set of values for 's' for which Laplace transform integral can be evaluated (i.e it converges)."

$x(t) = e^{2t}.u(t)$

Find $\mathcal{L}\{x(t)\} = ?$ ROC = ?

Sol^n

$$\mathcal{L}\{x(t)\} = X(s) = \int_{-\infty}^{\infty} x(t). e^{-st}\, dt$$

$$\Rightarrow \int_0^{\infty} e^{2t}. e^{-st}\, dt$$

$$\Rightarrow \int_0^{\infty} e^{-(s-2)t}\, dt$$

It converges for $s-2 > 0$ i.e $Re\{s\} > +2$

$$\Rightarrow \frac{-1}{s-2} \left| e^{-(s-2)t} \right|_0^{\infty}$$

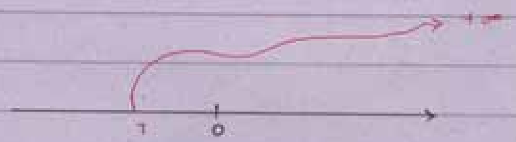
$$\Rightarrow \frac{1}{s-2}$$; ROC: $Re\{s\} > +2$.

-07

Types of Signals to Determine ROC

① Right Sided Signal:

(i.e start some where ; ends at $+\infty$)



② Left Sided Signal:

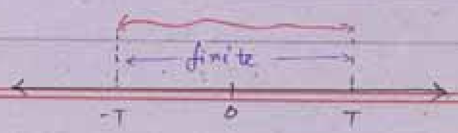
(i.e start some where ; ends at $-\infty$)

③ Two-Sided Signals:

(starts at $-\infty$; end at $+\infty$)

④ Finite-Duration Signals:

(Starts some where, end at finite interval)



-07 {ROC}

Rules for Region of Convergence:

① The ROC is always a region of S-plane to left or right of a vertical line or a strip b/w two lines.

② ROC never contains a Pole in it.

③ If $x(t)$ is right-sided fns then ROC is also right-sided.
i.e

$$Re\{s\} > a$$

where 'a' is right most-pole.

④ If $x(t)$ is left-side → ROC is also left-sided.

$$Re\{s\} < a$$

where 'a' is left-most pole.

⑤ If $x(t)$ is two sided or sum of left & right sided fns then ROC is either a strip ($a < Re\{s\} < b$) or else the individual ROC will not overlap producing null-set.

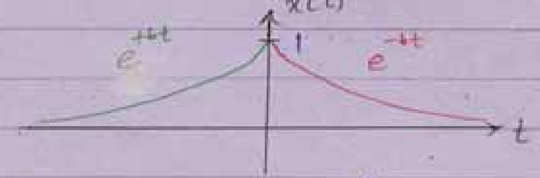
{Rule of ROC}

① If $x(t)$ is of finite duration then ROC is entire S-plane.

Que! For signal $x(t) = e^{-b|t|}$; $b>0$ find $\mathcal{L}\{x(t)\}$ and ROC

Sol

$$x(t) = e^{+bt}\,u(-t) + e^{-bt}\,u(t)$$



Now

$$X(s) = \int_{-\infty}^{0} e^{+bt}\,e^{-st}\,dt + \int_{0}^{\infty} e^{-bt}\,e^{-st}\,dt$$

$$X(s) = \int_{-\infty}^{0} e^{-(s-b)t}\,dt + \int_{0}^{\infty} e^{-(s+b)t}\,dt.$$

ROC : $s-b < 0$, $s < +b$, $Re\{s\} < b$ ROC : $s+b > 0$, $s > -b$, $Re\{s\} > -b$

$$= \frac{-1}{s-b}\left| e^{-(s-b)t} \right|_{-\infty}^{0} + \frac{(-1)}{s+b}\left| e^{-(s+b)t} \right|_{0}^{\infty}$$

$$= \frac{-1}{s-b}\left| e^{0} - e^{+(+\infty)} \right| + \frac{(-1)}{s+b}\left(e^{-\infty} - e^{0} \right|$$

$$X(s) = \frac{-1}{s-b} + \frac{1}{s+b}$$

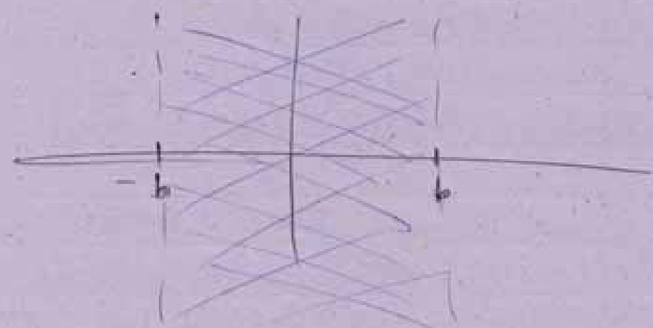
$$X(s) = \frac{1}{s+b} - \frac{1}{s-b}$$

$$= \frac{s-b-(s+b)}{(s+b)(s-b)}$$

$$X(s) = \frac{-2b}{s^2-b^2}$$

$$X(s) = \frac{-2b}{(s-b)(s+b)}$$

ROC:



ROC: $-b < Re\{s\} < b$

Properties of Laplace Transform

① **Linearity:**

If $\quad x_1(t) \xleftrightarrow{\mathcal{L}} X_1(s) \quad ; \text{ROC}: R_1$

and $\quad x_2(t) \xleftrightarrow{\mathcal{L}} X_2(s) \quad ; \text{ROC}: R_2$

then

$$a\,x_1(t) + b\,x_2(t) \xleftrightarrow{\mathcal{L}} a\,X_1(s) + b\,X_2(s)$$
$$; \; R_1 \cap R_2$$

Ex.1 Let $x(t) = x_1(t) + x_2(t)$

where

$$X_1(s) = \frac{1}{s+1} \quad \text{ROC}: \mathrm{Re}\{s\} > -1$$

and

$$X_2(s) = \frac{1}{(s+1)(s+2)} \quad \text{ROC}: \mathrm{Re}\{s\} > -1$$

Find $X(s)$ and its ROC.

Sol ROC for $X_1(s)$; ROC for $X_2(s)$

$$X(s) = X_1(s) + X_2(s)$$

$$X(s) = \frac{1}{s+1} + \frac{1}{(s+1)(s+2)}$$

$$X(s) \Rightarrow \frac{s+3}{(s+1)(s+2)} \quad ; \text{ROC}: \mathrm{Re}\{s\} > -1$$

② **Time - Shifting:** (Laplace ...)

If $\quad x(t) \xleftrightarrow{\mathcal{L}} X(s) \; ; \; \text{ROC} = R$

then

$$x(t-t_0) \longleftrightarrow X(s)\,e^{-st_0} \; ; \; \text{ROC} = R$$

③ **Time Scaling:**

If $\quad x(t) \xleftrightarrow{\mathcal{L}} X(s)$

then

$$x(at) \xleftrightarrow{\mathcal{L}} \frac{1}{|a|} X\left(\frac{s}{a}\right) \quad \text{ROC}: \frac{R}{a}$$

④ **Convolution in time - domain:**

If $\quad x_1(t) \xleftrightarrow{\mathcal{L}} X_1(s) \quad \text{ROC}: R_1$

and $\quad x_2(t) \longleftrightarrow X_2(s) \quad \text{ROC}: R_2$

then $\quad x_1(t) * x_2(t) \xleftrightarrow{\mathcal{L}} X_1(s) \cdot X_2(s)$

$$\text{ROC}: R_1 \cap R_2$$

Ex.1 Let $\quad X_1(s) = \dfrac{s+1}{s+2} \quad ; \mathrm{Re}\{s\} > -2$

and $\quad X_2(s) = \dfrac{s+2}{s+1} \quad ; \mathrm{Re}\{s\} > -1$

Find $\mathcal{L}\{x_1(t) * x_2(t)\}$

Sol

$$\mathcal{L}\{x_1(t) * x_2(t)\} = X_1(s) \cdot X_2(s) \quad \Big| \; \text{ROC}: -\infty < \mathrm{Re}\{s\} < \infty$$

$$= \frac{s+1}{s+2} \cdot \frac{s+2}{s+1} \quad \Big| \; \text{or ROC}:$$

$$\mathcal{L}\{x_1(t) * x_2(t)\} = 1 \quad \cancel{A \cdot 1} \quad \Big| \; \text{entire } s\text{-plane}$$

{Laplace Transform Properties}

(3) Differentiation in time-domain:

If $x(t) \xleftrightarrow{\mathcal{L}} X(s)$ ROC: R.

then $\frac{d}{dt} x(t) \xleftrightarrow{\mathcal{L}} s.X(s)$ ROC: Containing R

Table: 9.1 Page No. 691 [Oppenheim]
↳ Laplace transform Properties.

Inverse Laplace Transform

Mathematically,
If $x(t) \xleftrightarrow{\mathcal{L}} X(s)$

then $$x(t) = \frac{1}{2\pi j}\int_{\delta-j\infty}^{\delta+j\infty} X(s).\, e^{st}\, ds$$ where 'δ' is Real part of 'S'

This equation states that x(t) can be represented as a weighted integral of complex exponentials the Contour of integration is straight line in S-plane corresponding to all points satisfying the real value of 'S' is = 'δ'
$Re\{s\} = \delta$

{Laplace Inverse Transf}

For rational transforms we usually find inverse laplace by transform-pairs tables (Page 692).

Q. Find $x(t)$, if $X(s) = \frac{1}{(s+2)(s+1)}$
and
ROC: $Re\{s\} > -1$

Sol

$$\frac{1}{(s+2)(s+1)} = \frac{A}{s+2} + \frac{B}{s+1}$$

$$1 = A(s+1) + B(s+2)$$

let $s = -2$

$$1 = A(-2+1) + 0$$

$$\boxed{A = -1}$$

let $s = -1$

$$1 = A(-1+1) + B(-1+2)$$

$$\boxed{1 = B}$$

$$\frac{1}{(s+2)(s+1)} = \frac{-1}{s+2} + \frac{1}{s+1}$$

$$X(s) = \frac{1}{s+1} - \frac{1}{s+2}; \quad ROC = Re\{s\} > -1$$

* Using table of inv laplace transform

Pair 6: ⇒ $\frac{1}{s+\alpha}$; $Re\{s\} > -\alpha \rightarrow e^{-\alpha t}.u(t)$

Q2 Find $\mathcal{L}^{-1}\{X(s)\}$ if $X(s) = \frac{2s+1}{s^3+3s^2-4s}$; ROC : Re{s

(12-10-07)

$$X(s) = \frac{2s+1}{s(s^2+3s-4)}$$

$$X(s) = \frac{2s+1}{s(s+4)(s-1)}$$

Apply partial fraction

$$\frac{2s+1}{s(s+4)(s-1)} = \frac{A}{s} + \frac{B}{(s+4)} + \frac{C}{(s-1)}$$

$$A = \left\{ s \times \left| \frac{2s+1}{s(s+4)(s-1)} \right| \right\}_{s=}$$

$$A = -\frac{1}{4}$$

$$B = \left\{ (s+4) \frac{2s+1}{s(s+4)(s-1)} \right\}_{s=-4}$$

$$B = \frac{-8+1}{(-4)(-5)} = -\frac{7}{20}$$

$$C = \frac{2s+1}{s(s+4)} \Big|_{s=+1}$$

$$C = \frac{3}{5}$$

$$\frac{2s+1}{s(s+4)(s-1)} = \frac{-1/4}{s} + \frac{-7/20}{s+4}$$

$$= \frac{-1}{4s} - \frac{7}{20(s+4)}$$

$$= \frac{1}{20}\left(\frac{-5}{s} - \frac{7}{(s+4)} + \frac{12}{(s-}\right.$$

$$X(s) = \frac{1}{20}\left[\frac{-5}{s} - \frac{7}{(s+4)} + \frac{1}{s-}\right.$$

$\rightarrow$ Pair 6 $\frac{1}{s+\alpha}$; Re{s

$$x(t) = \frac{1}{20}\left[(-5) - 7e^{-4t} + 12\right.$$

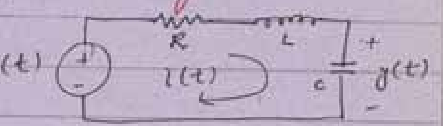
$$x(t) = \frac{12}{20} e^{t} u(t) - \frac{7}{20} e^{-4t} u$$

$$-\frac{1}{4}$$

$$x(t) = e^{-t}u(t) - e^{-2t}u(t)$$

$$x(t) = e^{-t}.u(t) - e^{-2t}\,u(t)$$

Find system response of following RLC ckt.



Applying KVL

$$x(t) = Ri(t) + v_L + y(t)$$

$$x(t) = Ri(t) + L\frac{di(t)}{dt} + y(t)$$

We have

$$i_c = C\frac{dv_c}{dt} = C\frac{dy(t)}{dt}$$

$$x(t) = RC\frac{dy(t)}{dt} + LC\frac{d^2}{dt^2}y(t) + y(t)$$

$$LC\frac{d^2y(t)}{dt^2} + RC\frac{dy(t)}{dt} + y(t) = x(t)$$

$$LC\,y''(t) + RC\,y'(t) + y(t) = x(t)$$

$$y''(t) + \frac{R}{L}y'(t) + \frac{1}{LC}y(t) = \frac{1}{LC}x(t)$$

Taking Laplace Transform of eq:

$$s^2Y(s) + \frac{R}{L}.sY(s) + \frac{1}{LC}Y(s) = \frac{1}{LC}X(s) \quad (1)$$

The System transfer fuc.

$$H(s) = \frac{Y(s)}{X(s)}$$

① $\Rightarrow$ $$Y(s)\left[s^2+\frac{R}{L}s+\frac{1}{LC}\right] = \frac{1}{LC}X(s)$$

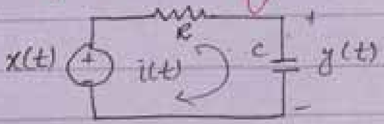
Hence,

$$\frac{Y(s)}{X(s)} = \frac{(1/LC)}{s^2+\frac{R}{L}s+\frac{1}{LC}}$$

A

$$H(s) = \frac{1/LC}{s^2+\frac{R}{L}s+\frac{1}{LC}}$$

Q: Find impulse Response of following circuits.



Applying KVL

$$Ri(t) + y(t) = x(t)$$

$$RC\frac{dy(t)}{dt} + y(t) = x(t)$$

$$RC\,y'(t) + y(t) = x(t)$$

$$y'(t) + \frac{1}{RC}y(t) = \frac{1}{RC}x(t)$$

$$sY(s) + \frac{1}{RC}Y(s) = \frac{1}{RC}X(s)$$

$$Y(s)\left[s+\frac{1}{RC}\right] = \frac{1}{RC}X(s)$$

$$\frac{Y(s)}{X(s)} = \frac{(1/RC)}{s+\frac{1}{RC}}$$

$$H(s) = \frac{1/RC}{s+\frac{1}{RC}}$$

Using table of inv - laplace,

$$h(t) = \left(\frac{1}{RC}\right)\cdot e^{-\left(\frac{1}{RC}\right)t}\cdot u(t)$$

$$h(t) = \frac{1}{\tau}e^{-\frac{t}{\tau}}\cdot u(t)$$

A

Q3: Find differential Equation model for this system if input; $x(t) = e^{-3t}\cdot u(t)$ is applied it produces an output; $y(t) = [e^{-t} - e^{-2t}]\cdot u(t)$.

Sol

$$x(t) = e^{-3t}\,u(t)$$

$$X(s) = \int_0^{\infty} e^{-3t}\,e^{-st}\,dt$$

$$X(s) = \int_0^{\infty} e^{-(s+3)t}\,dt$$

$$X(s) = \frac{1}{s+3} \quad \text{ROC: } Re\{s\} > -3$$

Similarly:

$$Y(s) = \frac{1}{(s+1)} - \frac{1}{(s+2)} \quad \text{ROC: } Re\{s\} > -1$$

The ratio

$$\frac{Y(s)}{X(s)} = \frac{\frac{(s+2)-(s+1)}{(s+1)(s+2)}}{\left(\frac{1}{s+3}\right)}$$

$$= \frac{(s+3)\left[s+2-s-1\right]}{(s+1)(s+2)}$$

$$H(s) = \frac{(s+3)}{(s+1)(s+2)}$$

$$Y(s)(s+1)(s+2) = X(s)(s+3)$$

$$\cancel{(sY(s)+Y(s))(sY(s)+2Y(s))} = sX(s)+3X(s)$$

$$Y(s)(s^2+3s+2) = sX(s)+3X(s)$$

$$s^2Y(s)+3s.Y(s)+2Y(s) = sX(s)+3X(s)$$

Take $\mathcal{L}^{-1}$;

$$y''(t)+3y'(t)+2y(t) = x'(t)+3x(t)$$

Discrete - Time Fourier Transform (DTFT)

To convert discrete time signals and system response into frequency domain we use DTFT.

It gives frequency spectrum of non-periodic discrete signals.

Recall for a C-T → FT

$$X(j\omega) = \int_{-\infty}^{\infty} x(t).e^{-j\omega t}\,dt$$

Similarly; for D-T domain this relationship becomes,

$$X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x[n].e^{-j\omega n}$$

So,

$$X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x[n]\,e^{-j\omega n}$$

Q1 :- for $x[n] = \{\underset{\uparrow}{0}, 1, 2, 3, 4\}$

find DTFT

Sol We have

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$$

$$X(e^{j\omega}) = (0)\, e^{-j\omega(0)} + (1)\, e^{-j\omega(1)} + (2) e^{-j\omega(2)} + 3 e$$

$$+ 4 e^{-j\omega(4)}$$

$$X(e^{j\omega}) = 0 + e^{-j\omega} + 2e^{-2j\omega} + 3e^{-3j\omega} + 4e^{-4j\omega}$$

A

Q2 Find fourier tranform of

$x[n] = \{1, 1, \underset{\uparrow}{1}, 1, 1\}$

Sol $$X(e^{j\omega}) = \sum_{n=-2}^{+2} x[n]\, e^{-j\omega n}$$

$$X(e^{j\omega}) = (1)\, e^{+j\omega(2)} + (1) e^{-j\omega(-1)} + (1)\, e^{-j\omega(0)} + (1) e$$

$$+ e^{-j\omega(2)}$$

$$X(e^{j\omega}) = e^{2j\omega} + e^{j\omega} + 1 + e^{-j\omega} + e^{-2}$$

Divide & multiply by (2)

$$2\left[\frac{e^{2j\omega} + e^{-2j\omega}}{2} + \frac{e^{j\omega} + e^{-j\omega}}{2} + \frac{1}{2}\right] \quad 2\cos 2\omega + 2\cos\omega + 1$$

Find $X(e^{j\omega})$ for signal $x[n] = a^n \cdot u[n]$; $a<1$

Sol we have,

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$$

Hence,

$$X(e^{j\omega}) = \sum_{n=0}^{\infty} a^n e^{-j\omega n}$$

$$X(e^{j\omega}) = \sum_{n=0}^{\infty} (ae^{-j\omega})^n$$

Here $(ae^{-j\omega})^n$ is an infinite Geometric series, hence using formula for G.S

$$X(e^{j\omega}) = \frac{1}{1 - ae^{-j\omega}}$$

Ans

Properties of DTFT

① **Periodicity**: In contrast to CTFT which in general is non-periodic, the DTFT is Periodic in ω-domain with Period '2π' i.e

$$X.\,e^{(j\omega+2\pi n)} = X.\,e^{j\omega}$$

② **Linearity**:

If $x_1[n] \xleftrightarrow{F} X_1(e^{j\omega})$

and

$x_2[n] \xleftrightarrow{F} X_2(e^{j\omega})$

then,

$$a x_1[n] + b x_2(n) \xleftrightarrow{F} a X_1 e^{j\omega} + b X_2 e^{j\omega}$$

③ **Time-Shifting**:

If

$x[n] \xleftrightarrow{F} X(e^{j\omega})$

then

$$x[n-n_0] \xleftrightarrow{F} X(e^{j\omega}).\,e^{-j\omega n_0}$$

Q1 (30/10/07) Let $x[n] = a^{|n|}$; $|a|<1$ find $X(e^{j\omega})$ for ...

Sol

We have

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n} \quad \text{——— ①}$$

also

$$x[n] = a^n . u[n] + a^{-n} u[-n]$$

① Becomes

$$X(e^{j\omega}) = \sum_{n=0}^{+\infty} a^n e^{-j\omega n} + \sum_{n=-\infty}^{-1} a^{-n} . e^{-j\omega n}$$

$$X(e^{j\omega}) = \sum_{n=0}^{\infty} (a e^{-j\omega})^n + \sum_{n=-\infty}^{-1} (a.e^{j\omega})^{-n}$$

Making substitution of variables in 2nd Summation -

$$m = -n$$

$$X(e^{j\omega}) = \sum_{n=0}^{\infty} (a e^{-j\omega})^n + \sum_{m=1}^{+\infty} (a.e^{j\omega})^{+m}$$

Note

The Fourier transform for D-T sig... in Continous in ω - domain.

{Properties of DTFT}

(4) Time Scaling (Time - expansion)

Time scaling in DT-domain takes on some what different form from its CT counter part $\left(\frac{1}{|a|} X\left(\frac{\omega}{a}\right)\right)$ but in DT

$x[\alpha n]$ doesn't represent scaling as $x[\alpha n]$ is not possible if $\alpha < 1$ Similarly $x[2n]$ is not time compression as $x[2n]$ consist of all even sample Hence instead of time scaling time expansion is defined in DT-domain as

If

$$x[n] \xleftrightarrow{F} X(e^{+j\omega})$$

then

$$x_{(k)}[n] \xleftrightarrow{F} X(e^{jk\omega})$$

where

$$x_{(k)}[n] = \begin{cases} x[n/k] & \text{If } k \text{ is multiple of } 'n' \\ 0 & \text{If } k \text{ is not multiple of } 'n'. \end{cases}$$

The Convolution:

In time domain:

If

$$x[n] \xleftrightarrow{F} X(e^{j\omega})$$

and

$$h[n] \xleftrightarrow{F} H(e^{j\omega})$$

then

$$x[n] * h[n] \xleftrightarrow{F} X(e^{j\omega}) . H(e^{j\omega})$$

SAMPLING

* Process of conversion of C-T signal into D-T signal.

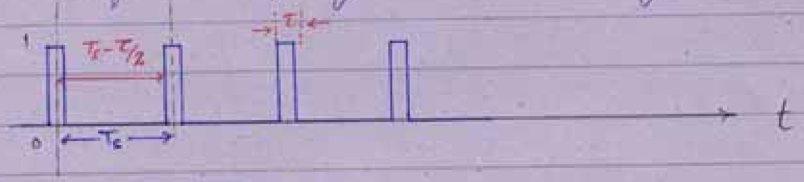
A C-T signal can be represented its samples to be processed and stored by discrete systems and storage. If a proper sampling rate used for sampling the C-T signal is reconstructed exactly as it was before sampling.

Mathematically sampled signal may be achieved by

$$x_s[t] = x(t) . p[t]$$

where $p[t]$ is a D-T signal called impulse train and is

The ideal impulse train is not generatable (Due to zero thickness) hence we use a signal for sampling denoted by $g_{T_0}(t)$



Using Fourier Series

$$g_{T_0}(t) = \begin{cases} 1 & -\tau/2 \le t \le \tau/2 \\ 0 & \tau/2 < t \le (T_s - \tau/2) \end{cases}$$

$$a_0 = \frac{1}{T} \int_T x(t)\, dt$$

$$a_0 = \frac{1}{T_s} \int_{-\tau/2}^{\tau/2} (1)\, dt$$

$$a_0 = \frac{1}{T_s} \left| t \right|_{-\tau/2}^{\tau/2} = \frac{1}{T_s}$$

$$a_0 = \frac{\tau}{T_s}$$

As the $g_{T_0}(t)$ is an Even function, hence

$$b_k = 0$$

$$a_k = \frac{2}{T_0} \int_{-\infty}^{\infty} x(t) \cdot \cos k\omega_s t\, dt$$

$$= \frac{2}{T_s} \int_{-\tau/2}^{+\tau/2} (1) \cdot \cos k\omega_s t\, dt$$

$$a_k = \frac{2}{T_s} \cdot \frac{1}{k\omega_s} \left| \sin k\omega_0 t \right|_{-\tau/2}^{+\tau/2}$$

$$= \frac{2}{T_s k\omega_s} \left[\sin \frac{k\omega_s \tau}{2} - \sin \frac{k\omega_s(-\tau)}{2} \right]$$

$$= \frac{2 \cdot 2}{T_s k\omega_s} \left[\sin \frac{k\omega_s \tau}{2} \right]$$

$$= \frac{2}{\frac{T_s k\omega_s}{2}} \left[\sin \frac{\omega_s k \tau}{2} \right] \times \frac{\tau}{\tau}$$

$$a_k = \frac{2\tau}{T_s} \times \frac{\left[\sin \frac{k\omega_s \tau}{2} \right]}{\frac{k\omega_s \tau}{2}}$$

$$a_k = \frac{2\tau}{T_s} \cdot \operatorname{sinc}\left(\frac{k\omega_s \tau}{2} \right)$$

{Sampling}

$$g_{T_0}(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos k\omega_0 t + \sum_{k=1}^{\infty} b_k \sin k\omega_0 t$$

$$g_{T_0}(t) = \frac{\tau}{T_s} + \frac{2\tau}{T_s}\sum_{k=1}^{\infty} \text{sinc}\left(\frac{k\omega_s\tau}{2}\right)\cdot \cos k\omega_s t.$$

as

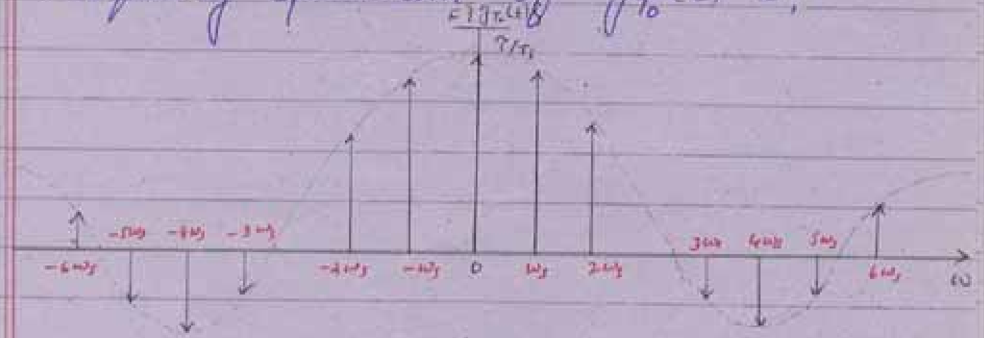
$$\cos\theta = \frac{e^{j\theta}+e^{-j\theta}}{2}$$

$$g_{T_0}(t) = \frac{\tau}{T_s} + \frac{2\tau}{T_s}\sum_{k=1}^{\infty} \text{sinc}\left(\frac{k\omega_s\tau}{2}\right)\left[\frac{e^{jk\omega_s t}+e^{jk(-\omega_s)t}}{2}\right]$$

$$g_{T_0}(t) = \frac{\tau}{T_s} + \frac{\tau}{T_s}\sum_{k=1}^{\infty} \text{Sinc}\left(\frac{k\omega_s\tau}{2}\right)\left[e^{jk\omega_s t}+e^{-jk\omega_s t}\right]$$

(Av. value at $\omega = 0$)

* Frequency spectrum of $g_{T_0}(t)$ is:



* Here we need help of "Frequency-shifting" property of Fourier Analysis which defines.

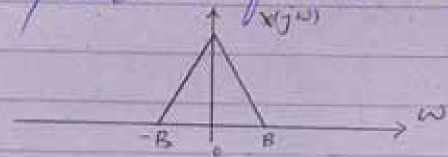
{Frequency-Shifting}

If $x(t) \longleftrightarrow X(j\omega)$

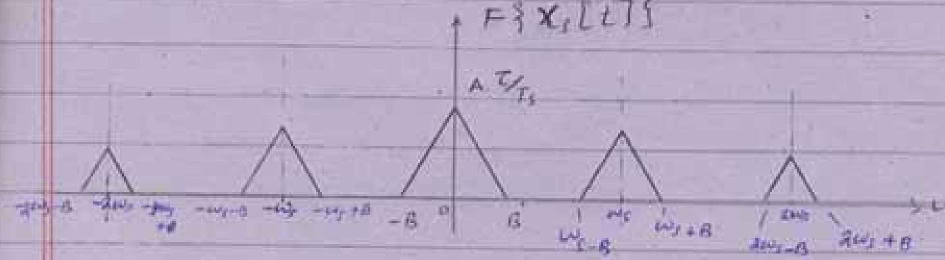
then $x(t)e^{+j\omega_0 t} \longleftrightarrow X(j\omega-\omega_0)$

* Now, spectrum of sampled signal, $x_s[t]$ will be,

Let, the spectrum of $x(t)$ i.e $X(j\omega)$ is.



Hence,

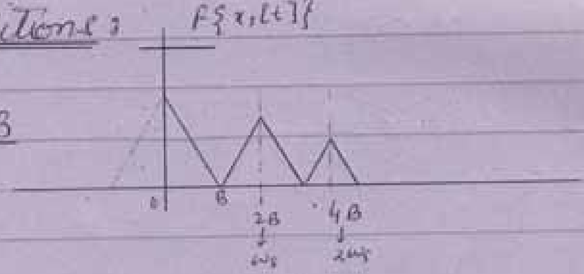


$$f_s \geq 2f_{max}$$

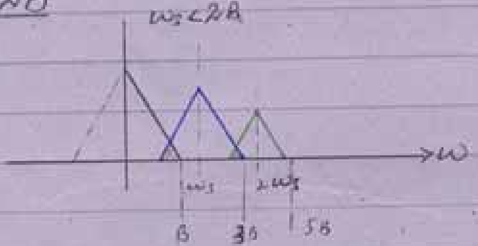
(Nyquist Theorem)

* Conditions:

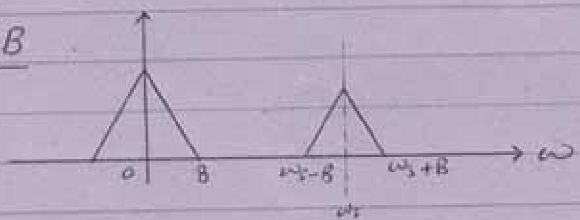
1. $\omega_s = 2B$



2. $\omega_s < 2B$



3. $\omega_s \gg 2B$



Sampling Theorem {Nyquist Theorem}

The sampling rate should be selected such that the sampling frequency $\omega_s \geq 2B$ where 'B' is bandwidth of information signal.

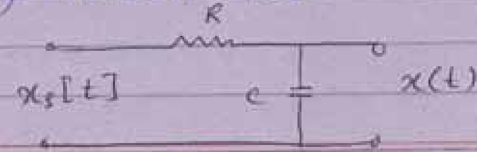
Reconstruction of C-T signal by Samples:

As the spectrum of sampled signal consist indicates ~~identical~~ that the spectrum of original spectrum of ~~x~~ $x(t)$ and repeated copies of its spectrum. if sample rate selected according to Nyquist Criteria i.e.

$$\omega_s \geq 2B$$

then base-band spectrum of $x(t)$ can be seperated by using Low-Pass filter.

A simple LPF may consists a capacitor & resistor as.



Effect of Under-Sampling

(ALIASING)

If sampling rate is less than then Nyquist Rate i.e

$$\omega_s < 2B$$

where 'B' is max frequency component in signal to be sampled (rad/sec)

The spectrum of information signal is no longer replicated in spectrum of sampled signal thus is no-longer recoverable by low pass filtering this effect in which the individual replicated spectrums overlap each other is referred to as (ALIASING). P: 530-31

Z - Transform

For some C-T signals of system response the Fourier transform may not exist, similarly for discrete time domain the DTFT for some functions may not exist.

The D-T counter part of Laplace Transform is Z-transform w/c exists for variety of D-T signals.

In C-T the Laplace Transform is defined as,

$$\mathcal{L}\{x(t)\} = X(s) = \int_{-\infty}^{\infty} x(t).e^{-st}\,dt$$

Similarly, the time-frequency relationship in Z-transform is defined as,

$$\mathcal{Z}\{x[n]\} = X(Z) = \sum_{n=-\infty}^{+\infty} x[n]\,Z^{-n}$$

where $Z = a + jb = re^{j\theta}$

Since Z-transform is an infinite power-series it exist only for those values of Z for w/c this series converges.

{Z-Transform}

ROC: ROC for Z-transform is defined as "Set of all values of Z for w/c $Z\{x(t)\}$ attains a finite value."

Q1 For $x[n] = \{\underset{\uparrow}{1}, 2, 5, 7, 0, 1\}$ find $X(Z)$

Sol

$$X(Z) = (1)Z^{-(0)} + (2)Z^{-1} + (5)Z^{-2} + (7)Z^{-3} + (0)Z^{-4} + (1)Z^{-5}$$

$$X(Z) = 1 + 2Z^{-1} + 5Z^{-2} + 7Z^{-3} + Z^{-5}$$

ROC: Entire Z-plane except $Z=0$.

Q2 For $x[n] = \{1, 2, \underset{\uparrow}{5}, 7, 0, 1\}$; Find $X(Z)$

$$X(Z) = (1)Z^{-(-2)} + (2)Z^{-(-1)} + 5Z^{-(0)} + 7(Z^{-1}) + (0)Z^{-2} + (1)Z^{-3}$$

$$= Z^{+2} + 2Z^{+1} + 5 + 7Z^{-1} + Z^{-3}$$

ROC: Entire Z-plane except $Z=0$, & $Z=\infty$.

Q3 For $x[n] = \delta[n]$. find $X(Z)$

Sol

$$x[n] = \{\ldots 0, 0, \underset{\uparrow}{1}, 0, 0, \ldots\}$$

$$X(Z) = \ldots + (0)Z^{-(-1)} + (1)Z^{-(0)} + 0Z^{-1} + \ldots$$

$$X(Z) = 1 \quad ; \quad \text{ROC : entire } Z\text{-plane.}$$

Q4: $x[n] = 2\delta[n-3]$, find $X(Z)$

$$x[n] = \{\ldots, 0, \underset{\uparrow}{0}, 0, 0, 2, 0, \ldots\}$$

$$X(Z) = (2)Z^{-(3)}$$
$$= 2Z^{-3}$$

ROC: Entire Z-plane except $Z=0$.

Q5 mixed:

Q6 For $x[n] = -a^n u[-n-1]$ find $X(Z)$ and ROC

Sol

$$X(Z) = \sum_{n=-\infty}^{\infty} (-a)^n u[-n-1] Z^{-n}$$

$$\Rightarrow \sum_{n=-\infty}^{+1} (-a)^n Z^{-n}$$

$$\Rightarrow -\sum_{n=-\infty}^{+1} (aZ^{-1})^n$$

changing limits

$$\Rightarrow -\sum_{n=-1}^{\infty} (a^{-1}Z)^n$$

Adding (+1) at $n=0$, $a^{-n}Z^{n} = -1$ and making limit form a zero.

$$X(Z) = 1 - \sum_{n=0}^{\infty} (a^{-1}Z)^{n}$$

∴ $a^{-n}Z^{n} < 1$ Condition

By Geo series form

$$\Rightarrow 1 - \frac{1}{1 - a^{-1}Z}$$

$$\Rightarrow \frac{\cancel{1} - a^{-1}Z - \cancel{1}}{1 - a^{-1}Z} \times \frac{aZ^{-1}}{aZ^{-1}}$$

$$\Rightarrow \frac{-1}{aZ^{-1} - 1} \quad --- ?$$

$$\Rightarrow \frac{1}{1 - aZ^{-1}} \times \frac{Z}{Z}$$

$$\Rightarrow \frac{Z}{Z - a} \;;$$

ROC:
So

$$a^{-1}Z < 1$$
$$Z < a$$
$$|Z| < a$$

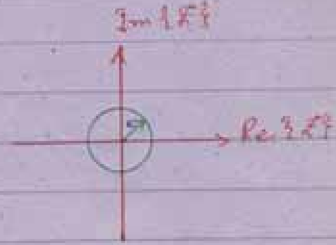
To Draw Region of Convergence

In Z-transform:

$$Z = a + jb$$

$$|Z| = \sqrt{a^2 + b^2}$$

$$c^2 = a^2 + b^2$$



Hence ROC of Z-transform is a circle at origin.

eg Q1: ROC: $|Z| > \frac{1}{2}$

Find $X(Z)$ for $x[n] = \left(\frac{1}{2}\right)^{n} u[n] + \left(\frac{-1}{3}\right)^{n} u[n]$

Sol

$$X(Z) = \sum_{n=-\infty}^{\infty} \left\{ \left(\frac{1}{2}\right)^{n} u[n] + \left(\frac{-1}{3}\right)^{n} u[n] \right\} Z^{-n}$$

$$X(Z) = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n} Z^{-n} + \sum_{n=0}^{\infty} \left(\frac{-1}{3}\right)^{n} Z^{-n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2} Z^{-1}\right)^{n} + \sum_{n=0}^{\infty} \left(\frac{-1}{3} Z^{-1}\right)^{n}$$

Using formula for infinite geometric series.

$$X(Z) = \frac{1}{1 - \frac{1}{2}Z^{-1}} + \frac{1}{1 + \frac{1}{3}Z^{-1}}$$

Q) Find $X(z)$ and draw ROC of

$$x[n] = \left(\frac{-1}{3}\right)^n u[n] - \left(\frac{1}{2}\right)^n u[-n+1]$$

$$X(z) = \sum_{n=-\infty}^{\infty} \left[\left(-\frac{1}{3}\right)^n u[n] - \left(\frac{1}{2}\right)^n u[-n+1] \right] z^{-n}$$

$$\Rightarrow \sum_{n=0}^{\infty} \left(\frac{-1}{3}\right)^n z^{-n} - \sum_{n=-\infty}^{-1} \left(\frac{1}{2}\right)^n z^{-n}$$

$$\Rightarrow \sum_{n=0}^{\infty} \left(\frac{-1}{3} z^{-1}\right)^n - \sum_{n=-\infty}^{-1} \left(\frac{1}{2} z^{-1}\right)^n$$

Inverting limits in 2nd part.

$$\Rightarrow \sum_{n=0}^{\infty} \left(\frac{-1}{3} z^{-1}\right)^n - \sum_{n=1}^{\infty} \left[\left(\frac{1}{2}\right)^{-1} z \right]^n$$

$$\Rightarrow \sum_{n=0}^{\infty} \left[\left(-\frac{1}{3}\right) z^{-1}\right]^n + 1 - \sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^{-1} z\right]^n$$

$$\Rightarrow \frac{1}{1 + \frac{1}{3} z^{-1}} + 1 - \frac{1}{1 - \left(\frac{1}{2}\right)^{-1} z}$$

$$\Rightarrow \frac{1}{1 + \frac{1}{3} z^{-1}} + \frac{\cancel{1} - \left(\frac{1}{2}\right)^{-1} z - \cancel{1}}{1 - \left(\frac{1}{2}\right)^{-1} z}$$

$$\Rightarrow \frac{1}{1 + \frac{1}{3} z^{-1}} + \frac{\left(\frac{1}{2}\right)^{-1} z}{\left(\frac{1}{2}\right)^{-1} z - 1} \times \frac{\frac{1}{2} z^{-1}}{\frac{1}{2} z^{-1}}$$

$$\Rightarrow \frac{1}{1 + \frac{1}{3} z^{-1}} + \frac{1}{1 - \frac{1}{2} z^{-1}}$$

$$\Rightarrow \frac{2z - 1/6}{(z - 1/2)(z + 1/3)}$$

$$X(z) = \frac{1 + \frac{1}{3} z^{-1} + 1 - \frac{1}{2} z^{-1}}{\left(1 - \frac{1}{2} z^{-1}\right)\left(1 + \frac{1}{3} z^{-1}\right)}$$

$$= \frac{2 - \frac{1}{6} z^{-1}}{\left(1 - \frac{1}{2} z^{-1}\right)\left(1 + \frac{1}{3} z^{-1}\right)} \quad b \ \frac{z}{z}$$

$$X(z) = \frac{2z - \frac{1}{6}}{\left(z - \frac{1}{2}\right)\left(z + \frac{1}{3}\right)}$$

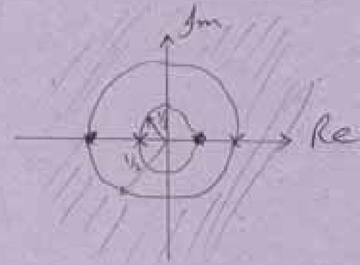
ROC

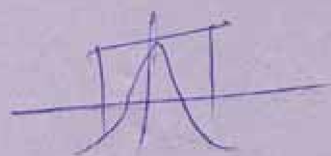
$\frac{1}{2} z^{-1} < 1$

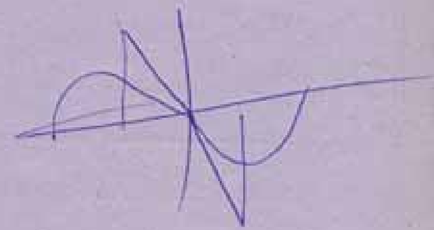
$\frac{1}{3} z^{-1} < 1$

Therefore

ROC $|z| > \frac{1}{2}$ & $|z| > -1/3$







$-\frac{1}{3} z^{-1} < 1$

$-\frac{1}{3} < |z|$

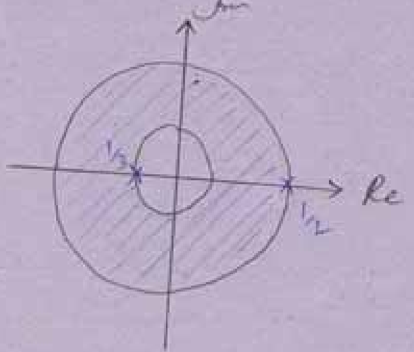
$\left(\frac{1}{2}\right)^{-1} z < 1$

$|z| < \frac{1}{2}$

ROC

$$X(z) = \frac{2z - 1/6}{(z+\frac{1}{3})(z-\frac{1}{2})}$$

ROC: $|z| > \frac{-1}{3}$ & $|z| < \frac{1}{2}$ Due to $u[-n+1]$



Properties of Z-transforms

① Linearity:

If
$$x_1[n] \xleftrightarrow{Z} X_1(z) \ ; \ ROC: R_{x_1}$$
and
$$x_2[n] \xleftrightarrow{Z} X_2(z) \ ; \ ROC: R_{x_2}$$
then
$$a x_1[n] \pm b x_2[n] \xleftrightarrow{Z} a X_1(z) \pm b X_2(z)$$
ROC: $R_{x_1} \cap R_{x_2}$

② Time-Shifting:

If
$$x[n] \xleftrightarrow{Z} X(z) \ ; \ ROC: R_x$$
then
$$x[n-n_0] \xleftrightarrow{Z} X(z) \cdot z^{-n_0}$$
ROC: R_x

③ Differentiation in Z-domain:

If
$$x[n] \xleftrightarrow{Z} X(z) \ ; \ ROC: R_x$$
then
$$x[n] \cdot z \longleftrightarrow \frac{dX(z)}{dz} \ ; \ ROC: R_x$$

④ Convolution in time-domain:

If
$$x_1[n] \xleftrightarrow{Z} X_1(z) \ ; \ ROC: R_{x_1}$$
$$x_2[n] \xleftrightarrow{Z} X_2(z) \ ; \ ROC: R_{x_2}$$
then
$$x_1[n] * x_2[n] \xleftrightarrow{Z} X_1(z) \cdot X_2(z)$$
ROC: $R_{x_1} \cap R_{x_2}$

Q1 Find output $y[n]$, of an LTI system if the impulse response $h[n]$ and input $x[n]$ are,

$$h[n] = \{\underset{\uparrow}{1}, -2, 1\}$$

and

$$x[n] = \begin{cases} 1 & 0 \le n \le 5 \\ 0 & \text{otherwise} \end{cases}$$

Sol

$$y[n] = x[n] * h[n]$$

$$Y(Z) = X(Z) \cdot H(Z)$$

$$H(Z) = (1) Z^{-0} + (-2) Z^{-1} + (1) Z^{-2}$$

$$= 1 - 2Z^{-1} + Z^{-2}$$

Similarly

$$X(Z) = 1 + Z^{-1} + Z^{-2} + Z^{-3} + Z^{-4} + Z^{-5}$$

Multiplying $X(Z)$ and $H(Z)$:

$$\begin{array}{rrrrrrrr}
1 & + Z^{-1} & + Z^{-2} & + Z^{-3} & + Z^{-4} & + Z^{-5} & & \\
 & & & 1 & - 2Z^{-1} & + Z^{-2} & & \times \\
\hline
 & & Z^{-2} & + Z^{-3} & + Z^{-4} & + Z^{-5} & + Z^{-6} & + Z^{-7} \\
 & -2Z^{-1} & -2Z^{-2} & -2Z^{-3} & -2Z^{-4} & -2Z^{-5} & -2Z^{-6} & \times \\
1 & + Z^{-1} & + Z^{-2} & + Z^{-3} & + Z^{-4} & + Z^{-5} & \times & \times \\
\hline
1 & - Z^{-1} & + 0 & + 0 & + 0 & + 0 & - Z^{-6} & + Z^{-7}
\end{array}$$

$$Y(Z) = 1 - Z^{-1} - Z^{-6} + Z^{-7}$$

Inversely:

$$y[n] = \{\underset{\uparrow}{1}, -1, 0, 0, 0, 0, -1, 1\}$$

Ans

Q2 If $x[n] = \{\underset{\uparrow}{1}, 3, -2\}$

and

$$h[n] = \{1, \underset{\uparrow}{2}, -1\}$$

Sol

$$X(Z) = 1 + 3Z^{-1} - 2Z^{-2}$$

$$H(Z) = Z^{+1} + 2 - Z^{-1}$$

$$\begin{array}{rrrrrr}
 & & - Z^{-1} & - 3Z^{-2} & + 2Z^{-3} & \times \\
 & 2 & + 6Z^{-1} & - 4Z^{-2} & \times & \\
Z^{+1} & + 3 & - 2Z^{-1} & \times & \times & \\
\hline
Z^{+1} & + 5 & + 3Z^{-1} & - 7Z^{-2} & + 2Z^{-3} &
\end{array}$$

Inversely:

$$y[n] = \{1, \underset{\uparrow}{5}, 3, -7, 2\}$$

H.W

Q3 $x[n] = \{1, 2, 0, -1, 1\}$; $h[n] = \{\underset{\uparrow}{1}, 3, -1, -2\}$

find $y[n]$ through convolution property.

Ans: $y[n] = \{1, 5, 5, -5, -6, 4, 1, -2\}$

(30-11-07) (Last Class)

Inverse - Z - transform by "LONG - DIVISION"

Q.1: Find $x[n]$, if $X(Z) = \dfrac{1}{1 - 1.5Z^{-1} + 0.5Z^{-1}}$; $|Z| > 1$

Sol:

$$1 - 1.5Z^{-1} + 0.5Z^{-1} \;\overline{\big)\; 1 \;}\; \big(1 + 1.5Z^{-1} + 1.75Z^{-2}$$

$$1 + 1.5Z^{-1} + 1.75Z^{-2}$$

$$1.5Z^{-1} - 0.5Z^{-2}$$

$$1.5Z^{-1} - 2.25Z^{-2} + 0.75Z^{-3}$$

$$1.75Z^{-2} - 0.75Z^{-3}$$

$$1.75Z^{-2} - 2.625Z^{-3} + 0.875Z^{-4}$$

Hence

$$X(Z) = 1 + 1.5Z^{-1} + 1.75Z^{-2} + \ldots\ldots$$

Inversly:

$$x[n] = \{ \underset{\uparrow}{1}, 1.5, 1.75, \ldots \}$$

Q2: Find $x[n]$; if $X(Z) = \dfrac{1}{1 - aZ^{-1}}$ $|Z| > a$

Sol: long division method:

$$1 - aZ^{-1} \;\overline{\big)\; 1 \;}\; \big(1 + aZ^{-1} + a^2Z^{-2} + a^3Z^{-3}$$

$$1 - aZ^{-1}$$

$$aZ^{-1}$$

$$aZ^{-1} - a^2Z^{-2}$$

$$a^2Z^{-2}$$

$$a^2Z^{-2} - a^3Z^{-3}$$

$$a^3Z^{-3}$$

$$a^3Z^{-3} - a^4Z^{-4}$$

$$a^4Z^{-4}$$

Hence

$$X(Z) = 1 + aZ^{-1} + a^2Z^{-2} + a^3Z^{-3} + a^4Z^{-4} \ldots$$

or

$$X(Z) = \sum_{n=0}^{\infty} a^n \cdot Z^{-n}$$

Inversly

$$x[n] = a^n \cdot u[n]$$

>>END<<

Random Variable :- → {Not Discussed in Course}
e.g Noise

↳ Will be discussed in Communication System :- Subject

Topics Covered:

- Signals & System
- Classification of Signal:
  1. Continous / discreate
  2. Digital / Analog
  3. Real / Complex
  4. Deterministic / Random
  5. Even / Odd
  6. Power / Energy
  7. Periodic / Aperiodic
- Basic operation on Signal (CT)
  1. Shift
  2. Reflection
  3. Scaling
- Complex Exponential & Sinusoidal Snl
  - (A) Real Exp Signal
    1. Growing real
    2. Decaying "
  - (B) Periodic Exp Snl
  - (C) General - complex exp Signal.
- Function Types (CT)
  1. Impulse
  2. Unit Step
  - + Relationship b/w δ & u.
- CT Systems.
  - ↳ LTI System
  - ↳ Response of LTI System.
- Properties of LTI System.
  - * Convolution.
- Response of LTI System or Complx Sinsoid
  - ↳ Eigen function
- Symmetry of waveform:
- Properties of F Transform.
- Properties of F Series.
- D-T Signals.
- D-T transformations.
- Types of LTI System.
  - * FIR
  - * IIR
- Types of DT Systems.
  - * Ideal
  - * Memoryless
  - * Linear
  - * Time Invarient.
- Basic Sequence
  - * Unit impulse
  - * Unit Step
  - * Exponential
  - * Periodicity.
- System Described by Diff Eqs.
- Laplace Transforms
  - ↳ ROC
  - ↳ Signal types to define ROC

- Properties of Laplace Transform
- Inverse Laplace
- D-T Fourier Transform:
- Properties of DTFT.
- Sampling
  - Conditions
  - Nyquist Theorem
  - ALIASING
- Z-transform
  - ROC
  - Properties.
  - To draw ROC.
- Inverse Z-transform
  - *
  - Long division.

Formulae

Energy Signal:

$$E = \lim_{L\to\infty} \int_{-L}^{L} |x(t)|^2 dt$$

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

Power Signal:

$$P = \lim_{L\to\infty} \frac{1}{2L} \int_{-L}^{L} |x(t)|^2 dt$$

$$P = \frac{1}{2N+1} \sum_{n=-\infty}^{\infty} |x[n]|^2$$

Impulse:

$$\delta[n] = u[n] - u[n-1]$$

$$\delta(t) = \frac{du(t)}{dt}$$

Unit Step:

$$u[n] = \sum_{m=0}^{\infty} \delta[n-m]$$

$$u(t) = \int_{0}^{\infty} \delta(t-\tau) d\tau$$

Convolution:

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(T-\tau) d\tau$$

Eigen: $H(j\omega) = \int_{-\infty}^{\infty} h(\tau) e^{-j\omega\tau} d\tau$

$$H(j\omega) = |H(j\omega)| . e^{\arg\{H(j\omega)\}}$$

Fourier Series: $\omega_0 = \frac{2\pi}{T_0}$

$$a_0 = \frac{1}{T} \int_T x(t) dt$$

$$a_k = \frac{2}{T} \int_T x(t) \cos k\omega_0 t\, dt$$

$$b_k = \frac{2}{T} \int_T x(t) \sin k\omega_0 t\, dt$$

Laplace Transform:

$$\mathcal{L}\{x(t)\} = X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt$$

Fourier Transform {C-T}:

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} dt$$

{D-T}

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

Sampling: $x_p[t] = x(t) . p[t]$

$$\omega_s \geq 2B$$

Z-transform:

$$Z\{x[n]\} = X(z) = \sum_{n=-\infty}^{+\infty} x[n] z^{-n}$$

$$|z| = \sqrt{a^2 + b^2}$$

Causal system:

$$a^{n-k}\, u[n-k] = \frac{z^{-k}}{1 - az^{-k}}$$

Anti-Causal:

$$-a^n . u[-n-1] = \frac{1}{1 - az^{-1}}$$

Properties

* LTI System :

(1) Memoryless (2) Causality (3) Commutativity
(4) Stability (5) Associativity (6) Distributivity
(7) Convolution

* Fourier Transform : (CT)

(1) Linearity:

$$x(t) \xleftrightarrow{F.T} X(j\omega) \; ; \; y(t) \xleftrightarrow{F.T} Y(j\omega)$$

$$Ax(t) + By(t) \xleftrightarrow{F.T} AX(j\omega) + BY(j\omega)$$

(2) Time Shifting:

$$x(t) \xleftrightarrow{F.T} X(j\omega)$$

$$x(t-t_0) \xleftrightarrow{F.T} X(j\omega)\, e^{-j\omega t_0}$$

(3) Time Scaling:

$$x(t) \xleftrightarrow{F.T} X(j\omega)$$

$$x(\alpha t) \longleftrightarrow \frac{1}{|\alpha|} \cdot X\left(\frac{j\omega}{\alpha}\right)$$

(4) Convolution:

$$x(t) \longleftrightarrow X(j\omega) \; ; \; h(t) \longleftrightarrow H(j\omega)$$

$$y(t) \Rightarrow x(t) * h(t) \longleftrightarrow X(j\omega) \cdot H(j\omega) \Rightarrow Y(j\omega)$$

(5) Duality:

$$x(t) \longleftrightarrow X(j\omega) \; ; \; X(t) \longleftrightarrow x(j\omega)$$

(6) Differentiation in t-domain:

$$x(t) \longleftrightarrow X(j\omega) \; ; \; \frac{d}{dt} x(t) \longleftrightarrow j\omega . X$$

* Fourier Series : (CT)

(1) Linearity:

$$z(t) = Ax(t) + By(t) \xleftrightarrow{FS} c_k \Rightarrow Aa_k + Bb_k$$

(2) Time Shifting:

$$x(t-t_0) \xleftrightarrow{FS} e^{-jk\omega_0 t_0} a_k$$

{C-T Fourier Series}

(3) Time Reversal:

$$x(t) \xleftrightarrow{FS} a_k \; ; \; x(-t) \xleftrightarrow{FS} a_{-k}$$

(4) Time Scaling:

$$x(\alpha t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk(\alpha\omega_0)t}$$

* Laplace Transform :

(1) Linearity:

$$x_1(t) \xleftrightarrow{\mathcal{L}} X_1(s) \; ; \; x_2(t) \xleftrightarrow{\mathcal{L}} X_2(s) \; ;; \; ROC: R_1, ROC: R_2$$

$$ax_1(t) + bx_2(t) \xleftrightarrow{\mathcal{L}} aX_1(s) + bX_2(s) \; ; \; R_1 \cap R_2$$

(2) Time Shifting:

$$x(t-t_0) \xleftrightarrow{\mathcal{L}} X(s)\, e^{-st_0} \; ; \; ROC = R$$

(3) Time Scaling:

$$x(\alpha t) \xleftrightarrow{\mathcal{L}} \frac{1}{|\alpha|} X\left(\frac{s}{\alpha}\right) \; ; \; ROC = \frac{R}{\alpha}$$

(4) Convolution:

$$x_1(t) * x_2(t) \xleftrightarrow{\mathcal{L}} X_1(s) \cdot X_2(s) \; ; \; R_1 \cap R_2$$

(5) Differentiation:

$$\frac{d}{dt} x(t) \xleftrightarrow{\mathcal{L}} s . X(s) \quad ROC: \text{Containing } R.$$

* Fourier Transform : (DT)

(1) Periodicity:

$$X . e^{(j\omega + 2\pi n)} = X . e^{j\omega}$$

(2) Linearity:

$$ax_1[n] + bx_2[n] \xleftrightarrow{F} aX_1 e^{j\omega} + bX_2 e^{j\omega}$$

(3) Time Shifting:

$$x[n-n_0] \xleftrightarrow{F} X(e^{j\omega})\, e^{-j\omega n_0}$$

{DTFT}

④ Time Scaling (Expension):

$x[n] \xleftrightarrow{F} X e^{j\omega}$; $x_k[n] \xleftrightarrow{F} X . e^{jk\omega}$

where

$$x_{(k)}[n] = \begin{cases} x\left[\frac{n}{k}\right] & \text{If } k \text{ multiple of} \\ 0 & k \text{ not multiple of} \end{cases}$$

⑤ Convolution:

$x[n] \xleftrightarrow{F} X(e^{j\omega})$; $h[n] \xleftrightarrow{F} H(e^{j\omega})$

then $x[n] * h[n] \xleftrightarrow{F} X(e^{j\omega}) . H(e^{j\omega})$

Z - transforms:

① Linearity:

$x_1[n] \xleftrightarrow{Z} X_1(Z)$; ROC : Rx_1

$x_2[n] \xleftrightarrow{Z} X_2(Z)$; ROC : Rx_2

then

$a x_1[n] \pm b x_2[n] \xleftrightarrow{Z} a X_1(Z) \pm b X_2(Z)$

$RX_1 \cap RX_2$

② Time - Shifting:

$x[n - n_0] \xleftrightarrow{Z} X(Z) . Z^{-n_0}$ :: RX

③ Differentiation:

$x[n] \xleftrightarrow{Z} \frac{d X(Z)}{dZ}$ :: RX

④ Convolution:

$x_1[n] * x_2[n] \xleftrightarrow{Z} X_1(Z) . X_2(Z)$

ROC : $RX_1 \cap RX_2$